ME451: Control Systems

Lecture 8
Modeling of DC motors

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Course roadmap

Modeling
- Laplace transform
- Transfer function
- Models for systems
  - electrical
  - mechanical
  - electromechanical
- Block diagrams
- Linearization

Analysis
- Time response
  - Transient
  - Steady state
- Frequency response
  - Bode plot
- Stability
  - Routh-Hurwitz
  - Nyquist

Design
- Design specs
- Root locus
- Frequency domain
- PID & Lead-lag
- Design examples

What is DC motor?
An actuator, converting electrical energy into rotational mechanical energy

(You will see DC motor during Lab 1 and 4.)

Why DC motor?

- Advantages:
  - high torque
  - speed controllability
  - portability, etc.

- Widely used in control applications: robot, tape drives, printers, machine tool industries, radar tracking system, etc.

- Used for moving loads when
  - Rapid (microseconds) response is not required
  - Relatively low power is required
How does DC motor work?

(from Dorf and Bishop book)

Model of DC motor

Modeling of DC motor: time domain

- Armature circuit \( e_a(t) = R_a i_a(t) + L_a \frac{d i_a(t)}{dt} + e_b(t) \)
- Connection between mechanical/electrical parts
  - Motor torque \( \tau(t) = K_\tau i_a(t) \)
  - Back EMF \( e_b(t) = K_b \omega(t) \)
- Mechanical load \( J\Dot{\theta}(t) = \tau(t) - B\Dot{\theta}(t) - \tau_l(t) \)
- Angular position \( \omega(t) = \Dot{\theta}(t) \)

Modeling of DC motor: s-domain

- Armature circuit \( I_a(s) = \frac{1}{R_a + L_a s} (E_a(s) - E_b(s)) \)
- Connection between mechanical/electrical parts
  - Motor torque \( T(s) = K_\tau I_a(s) \)
  - Back EMF \( E_b(s) = K_b \Omega(s) \)
- Mechanical load \( \Omega(s) = \frac{1}{J s + B} (T(s) - T_L(s)) \)
- Angular position \( \Theta(s) = \frac{1}{s} \Omega(s) \)
DC motor: Block diagram

Ex: Derivation of transfer functions

Useful formula for feedback

DC motor: Transfer functions (TF)
DC motor: Transfer functions (cont’d)

Note: In many cases $L_a \ll R_a$. Then, an approximated TF is obtained by setting $L_a = 0$.

\[
\frac{\Omega(s)}{E_a(s)} = \frac{K_T}{(L_a s + R_a)(J_s + B) + K_b K_T} \approx \frac{K_T}{R_a(J_s + B) + K_b K_T}
\]

\[
= \frac{K}{T_s + 1} \left( K := \frac{K_T}{R_a B + K_b K_T}, T = \frac{R_a J}{R_a B + K_b K_T} \right)
\]

$2^{nd}$ order system $\Rightarrow$ $1^{st}$ order system

Summary and Exercises

- Modeling of DC motor
  - What is DC motor and how does it work?
  - Derivation of a transfer function
  - Block diagram with feedback

- Next
  - Stability of linear control systems, one of the most important topics in feedback control

- Exercises
  Go over the derivation for DC motor transfer functions by yourself. Obtain $T(s)/E_a(s)$.

Main message until this point

Many systems can be represented as transfer functions!

Using the transfer functions, .... (to be continued)