ME451: Control Systems

Lecture 23
Bode diagram of simple systems

Dr. Jongeun Choi
Department of Mechanical Engineering
Michigan State University

Course roadmap

Modeling
- ✓ Laplace transform
- ✓ Transfer function
- ✓ Models for systems
  - ✓ electrical
  - ✓ mechanical
  - ✓ electromechanical
- ✓ Block diagrams
- ✓ Linearization

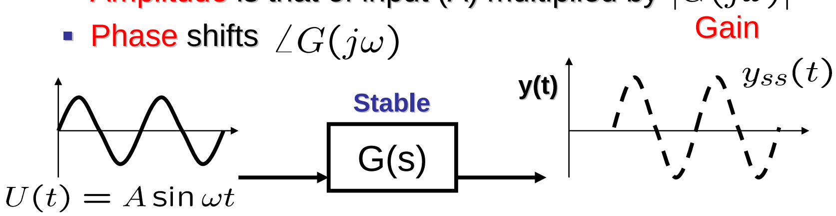
Analysis
- ✓ Time response
  - ✓ Transient
  - ✓ Steady state
- ✓ Frequency response
  - Bode plot
- ✓ Stability
  - ✓ Routh-Hurwitz
  - Nyquist

Design
- Design specs
- ✓ Root locus
- Frequency domain
- ✓ PID & Lead-lag
- Design examples

(Matlab simulations &) laboratories

Frequency response (review)

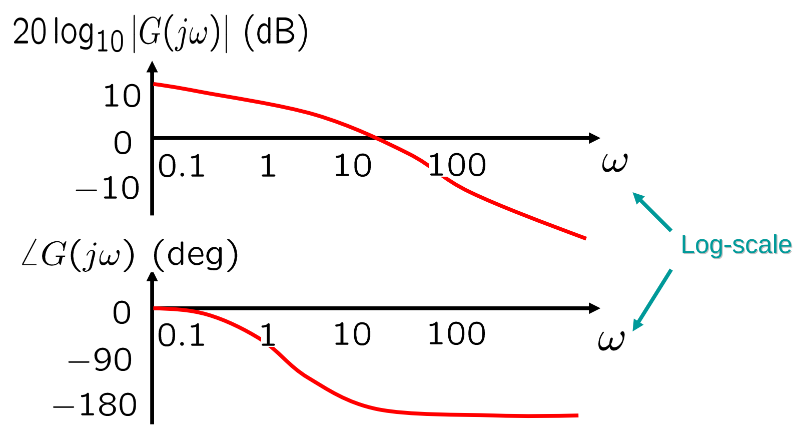
- Steady state output $y_{ss}(t) = A|G(j\omega)|\sin(\omega t + \angle G(j\omega))$
  - Frequency is same as the input frequency ω
  - Amplitude is that of input (A) multiplied by $|G(j\omega)|$
  - Phase shifts $\angle G(j\omega)$



- Frequency response function (FRF): G(jω)
- Bode plot: Graphical representation of G(jω)

Bode plot of G(jω) (review)

- Bode plot consists of gain plot & phase plot



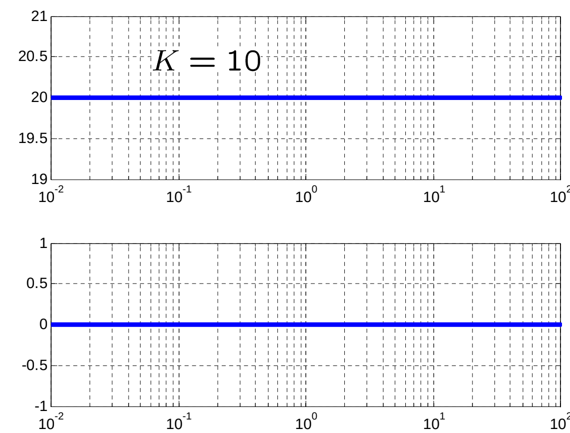
Sketching Bode plot

- Basic functions (Today)
  - Constant gain
  - Differentiator and integrator
  - Double integrator
  - First order system and its inverse
  - Second order system
  - Time delay
- Product of basic functions (Next lecture)
  1. Sketch Bode plot of each factor, and
  2. Add the Bode plots graphically.

Main advantage of Bode plot!

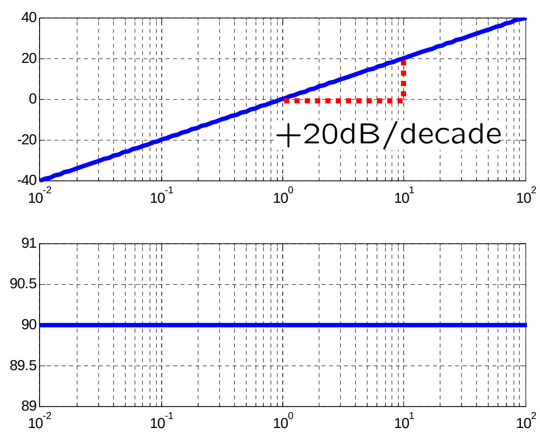
Bode plot of a constant gain

- TF $G(s) = K \Rightarrow |G(j\omega)| = K,\ \angle G(j\omega) = 0^{\circ},\ \forall\omega$



Bode plot of a differentiator

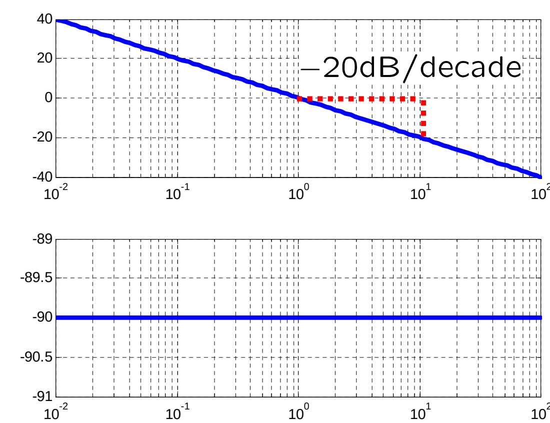
- TF $G(s) = s \Rightarrow |G(j\omega)| = \omega,\ \angle G(j\omega) = 90^{\circ},\ \forall\omega$



Bode plot of an integrator

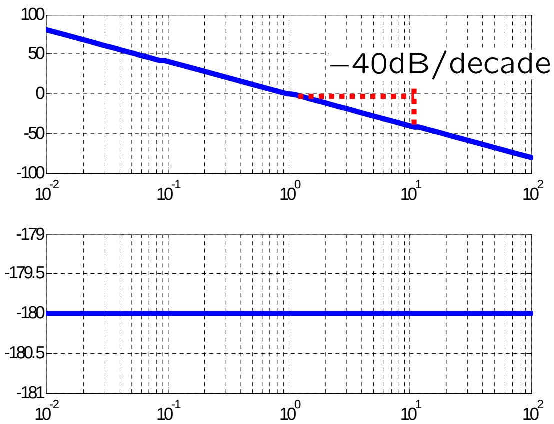
- TF $G(s) = \frac{1}{s} \Rightarrow |G(j\omega)| = \frac{1}{\omega},\ \angle G(j\omega) = -90^{\circ},\ \forall\omega$

Mirror image of the bode plot of 1/s with respect to ω-axis.



Bode plot of a double integrator

- TF $G(s) = \frac{1}{s^2} \Rightarrow |G(j\omega)| = \frac{1}{\omega^2},\ \angle G(j\omega) = -180^\circ,\ \forall\omega$



Bode plot of a 1st order system

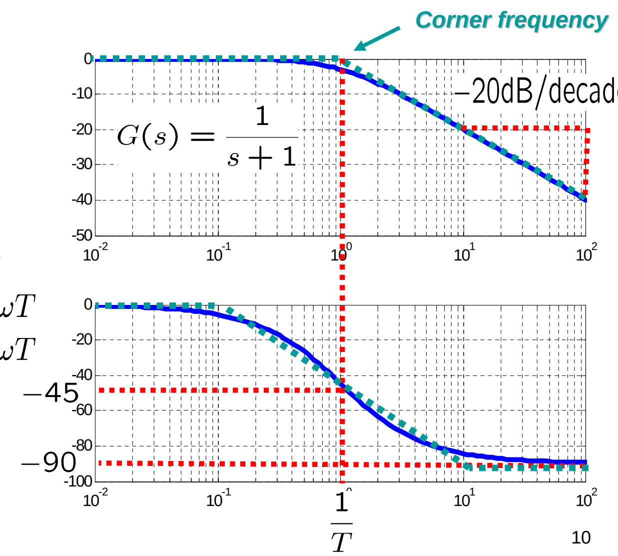
- TF

$$G(s) = \frac{1}{Ts+1}$$

$$G(j\omega) = \frac{1}{j\omega T + 1}$$

$$\approx \begin{cases} 1 & \text{if } 1 \gg \omega T \\ \frac{1}{j\omega T} & \text{if } 1 \ll \omega T \end{cases}$$

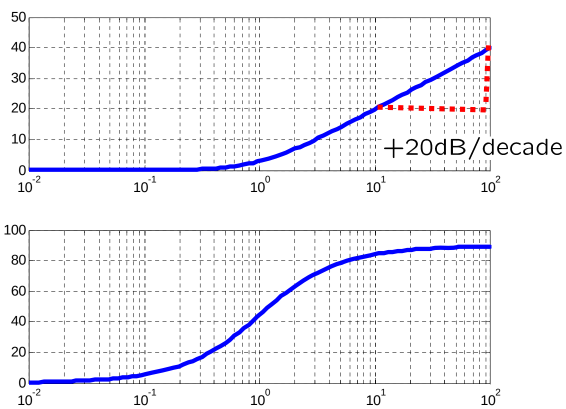
Straight line approximation



Bode plot of an inverse system

- TF $G(s) = Ts + 1 = \left(\frac{1}{Ts+1}\right)^{-1}$

Mirror image of the original bode plot with respect to ω-axis.



Bode plot of a 2nd order system

- TF

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

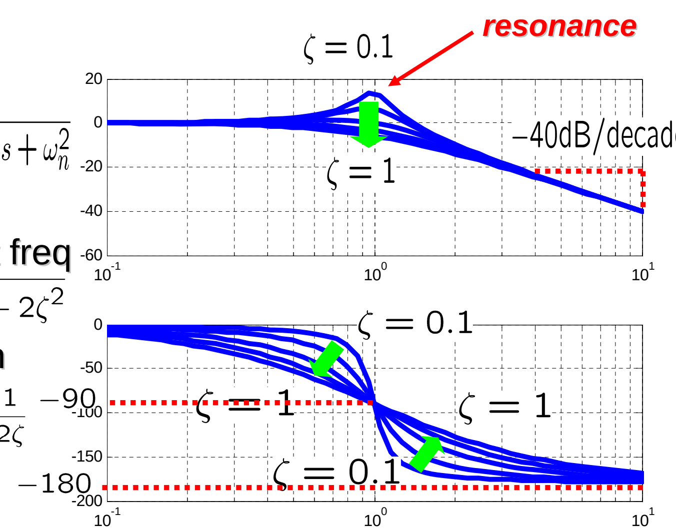
- Resonant freq

$$\omega_r := \omega_n\sqrt{1 - 2\zeta^2}$$

- Peak gain

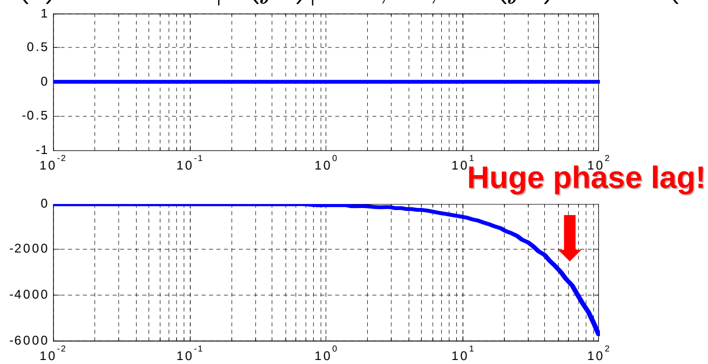
$$\frac{1}{2\zeta\sqrt{1-\zeta^2}} \approx \frac{1}{2\zeta}$$

if ζ is small



Bode plot of a time delay

- TF $G(s) = e^{-Ts} \Rightarrow |G(j\omega)| = 1, \forall\omega,\ \angle G(j\omega) = -\omega T(\text{rad})$



As can be explained with Nyquist stability criterion, this phase lag causes instability of the closed-loop system, and hence, the difficulty in control.

Remark

- Use Matlab "bode.m" to obtain precise shape.
- ALWAYS check the correctness of
  - Low frequency gain (DC gain) $G(0)$
  - High frequency gain $G(\infty)$
- Example $G(s) = \dfrac{10(s+1)}{s+5}$

Exercises

- Sketch bode plot.

$G(s) = 1$ $\quad G(s) = 0.1$ $\quad G(s) = -10$

$G(s) = s^2$ $\quad G(s) = s^3$ $\quad G(s) = \dfrac{1}{s^3}$

$G(s) = \dfrac{1}{10s+1}$ $\quad G(s) = \dfrac{10}{s+10}$ $\quad G(s) = 10s+1$

$G(s) = 2s+1$ $\quad G(s) = \dfrac{s}{5}+1$ $\quad G(s) = \dfrac{4}{s^2+2s+4}$

Summary

- Bode plot of various simple transfer functions.
  - Constant gain
  - Differentiator, integrator
  - 1st order and 2nd order systems
  - Time delay
- Sketching Bode plot is just ….
  - to get a rough idea of the characteristic of a system.
  - to interpret the result obtained from computer.
  - to detect erroneous result from computer.
- Next, Bode plot of connected systems