ME451: Control Systems

Lecture 21
Root locus: Lag compensator & Lead-lag compensator design

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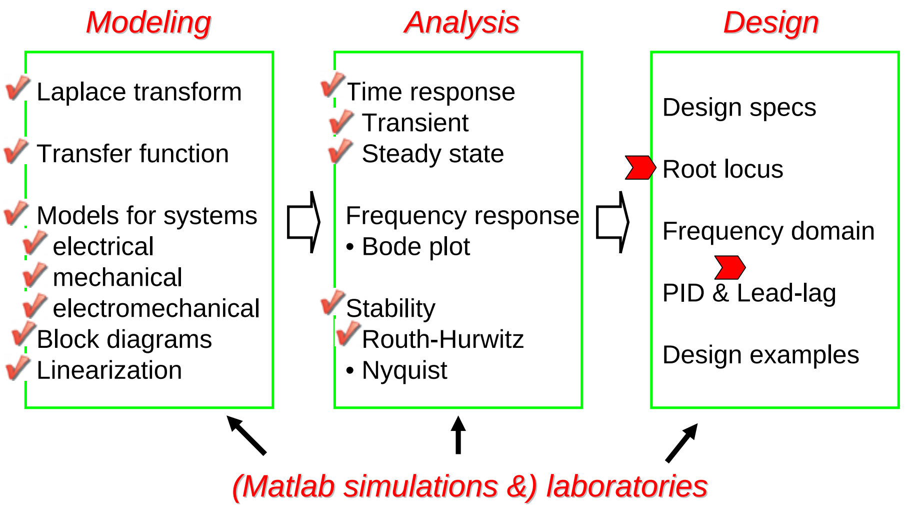
Course roadmap



Modeling

- ✓ Laplace transform
- ✓ Transfer function
- ✓ Models for systems
  - ✓ electrical
  - ✓ mechanical
  - ✓ electromechanical
- ✓ Block diagrams
- ✓ Linearization

Analysis

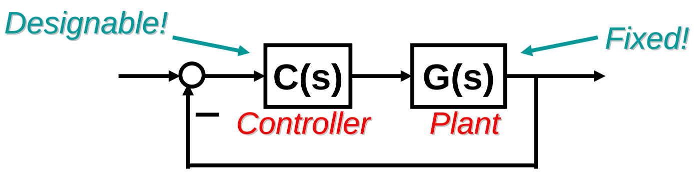
- ✓ Time response
  - ✓ Transient
  - ✓ Steady state
- Frequency response
  - • Bode plot
- ✓ Stability
  - ✓ Routh-Hurwitz
  - • Nyquist

Design

- Design specs
- ➤ Root locus
- Frequency domain
- ➤ PID & Lead-lag
- Design examples

(Matlab simulations &) laboratories

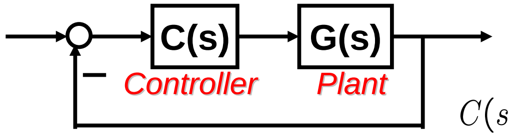
Closed-loop design by root locus



- Place closed-loop poles at desired location
  - by tuning the gain C(s)=K.
- If root locus does not pass the desired location, then reshape the root locus
  - by adding poles/zeros to C(s).

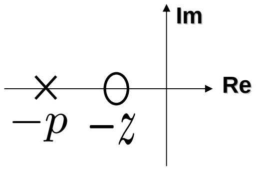
Compensation

Lead and lag compensators (review)

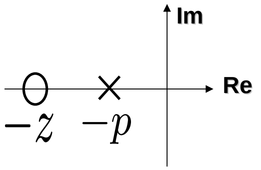


$$C(s) = K\frac{s+z}{s+p}, \ (z>0, p>0)$$

- Lead compensator



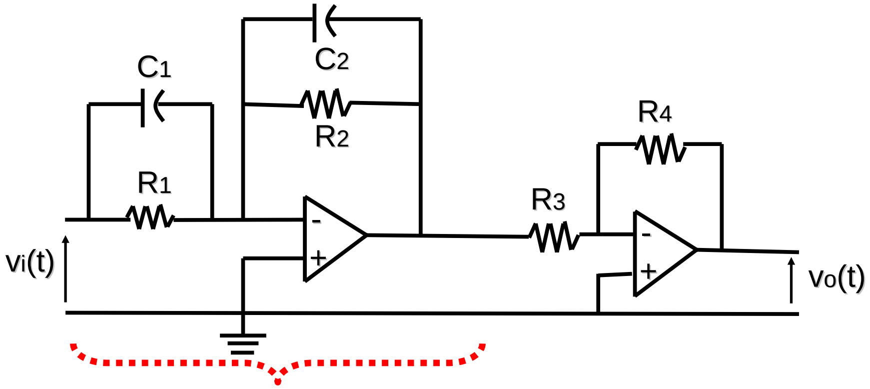
- Lag compensator



The reason why these are called "lead" and "lag" will be explained in frequency response approach (later in this course).

Compensator realization

- One example, using operational amplifiers



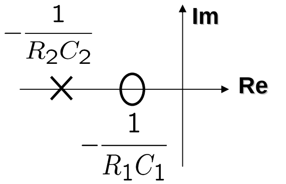
$$-\frac{R_2}{R_1}\cdot\frac{R_1C_1s+1}{R_2C_2s+1}$$

Compensator realization (cont'd)

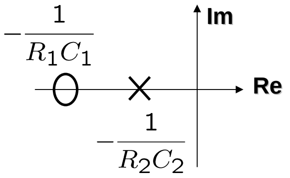
- Transfer function

$$C(s)=\frac{V_o(s)}{V_i(s)}=\frac{R_2R_1C_1(s+\frac{1}{R_1C_1})}{R_1R_2C_2(s+\frac{1}{R_2C_2})}\cdot\frac{R_4}{R_3}=\overset{K}{\boxed{\frac{R_4C_1}{R_3C_2}}}\cdot\frac{s+\overset{z}{\boxed{\frac{1}{R_1C_1}}}}{s+\underset{p}{\boxed{\frac{1}{R_2C_2}}}}$$

- Lead compensator



- Lag compensator



Roles of lead and lag compensators

- Lead compensator (Done)
  - Improve transient response
  - Improve stability

$$C_{Lead}(s)=K_1\frac{s+z_1}{s+p_1}$$

- Lag compensator (Today)
  - Reduce steady state error

$$C_{Lag}(s)=K_2\frac{s+z_2}{s+p_2}$$

- Lead-lag compensator (Today)
  - Take into account all the above issues.

$$C_{LL}(s)=C_{Lead}(s)C_{Lag}(s)$$

Radar tracking system

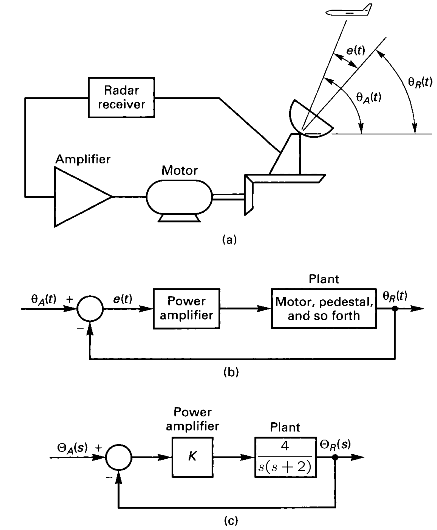
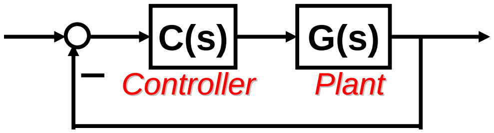


Figure 7.1 Radar tracking system.

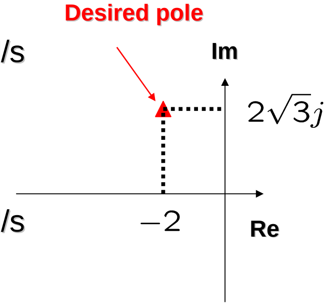
Lead-lag compensator design

- Consider a system

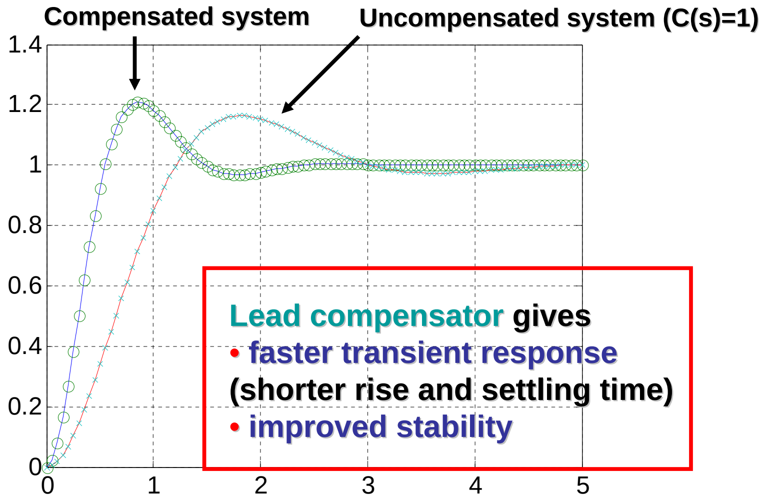
$$G(s)=\frac{4}{s(s+2)}$$



- Analysis of CL system for C(s)=1
  - Damping ratio ζ=0.5
  - Undamped natural freq. ω_n=2 rad/s
  - Ramp-error constant Kv=2
- Performance specification
  - Damping ratio ζ=0.5
  - Undamped natural freq. ω_n=4 rad/s
  - Ramp-error constant Kv=50



Comparison of step responses (after lead compensation)



Lead compensator gives
- **faster transient response**
(shorter rise and settling time)
- **improved stability**

Error constants (after lead compensation)

$$G(s)C_{Lead}(s)=\frac{4}{s(s+2)}\cdot\frac{4.675(s+2.9)}{s+5.4}$$

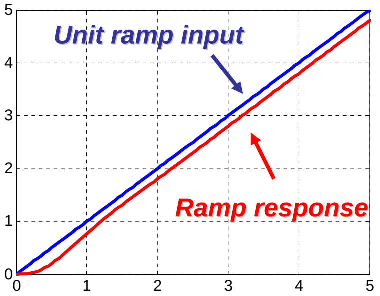
- Step-error constant

$$K_p:=\lim_{s\to 0}G(s)C_{lead}(s)=\infty$$

- Ramp-error constant

$$K_v:=\lim_{s\to 0}sG(s)C_{lead}(s)=5.02$$

NOT SATISFACTORY!



Lag compensator can reduce steady-state error.

How to design lag compensator

- Lag compensator $C_{Lag}(s)=\dfrac{s+z}{s+p}$
- We want to increase ramp-error constant

$$K_v=\lim_{s\to 0}sG(s)C_{Lead}(s)C_{Lag}(s)=5.02\cdot\frac{z}{p}>50$$

Take, for example, z=10p.

- We do not want to change CL pole location s_1 so much (already satisfactory transient).

$$\left.\begin{array}{r}1+G(s_1)C_{Lead}(s_1)=0\\ C_{Lag}(s_1)\approx 1\end{array}\right\}\Rightarrow 1+G(s_1)C_{Lead}(s_1)C_{Lag}(s_1)\approx 0$$

Guidelines to choose z and p

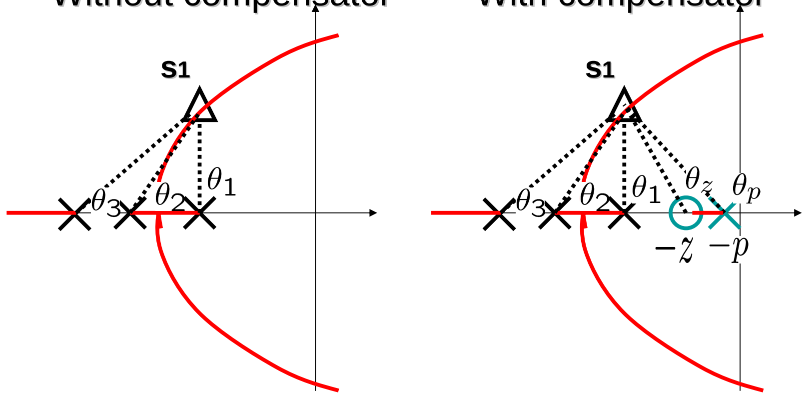
- The zero and the pole of a lag compensator should be close to each other, for

$$C_{Lag}(s_1) \approx 1$$

- The pole of a lag compensator should be close to the origin, to have a large ratio z/p, leading to a large ramp-error constant Kv.
- However, the pole of a lag compensator too close to the origin may be problematic:
  - Difficult to realize (recall op-amp realization)
  - Slow settling (due to closed-loop pole near the origin)

Root locus with lag compensator

- Without compensator
- With compensator



$$-\theta_1-\theta_2-\theta_3=180 \qquad -\theta_1-\theta_2-\theta_3+\theta_z-\theta_p\approx 180$$

How to design lag compensator

- For the desired CL pole $s_1=-2+2\sqrt{3}j$

$$\boxed{C_{Lag}(s_1)\approx 1 \Leftrightarrow \left|\frac{s_1+10p}{s_1+p}\right|\approx 1 \quad \angle\left(\frac{s_1+10p}{s_1+p}\right)\approx 0}$$

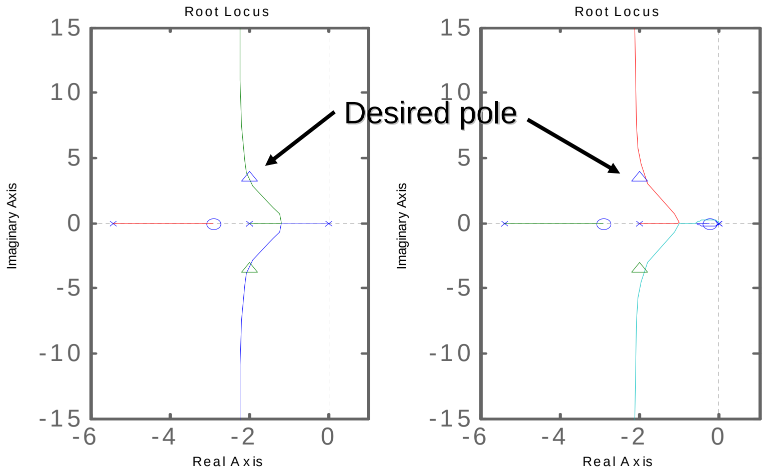
- Take a small p (by trial-and-error!)

$$p=0.025 \Rightarrow \left|\frac{s_1+10p}{s_1+p}\right|=0.97 \quad \angle\left(\frac{s_1+10p}{s_1+p}\right)\approx -2.88^o$$

- Lead-lag controller

$$\boxed{C_{LL}(s)=4.675\frac{s+2.9}{s+5.4}\cdot\frac{s+0.25}{s+0.025}}$$

Root locus

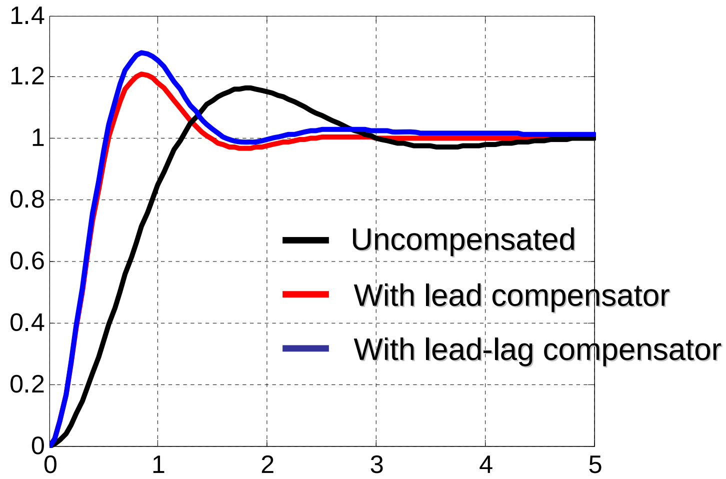


With lead compensator

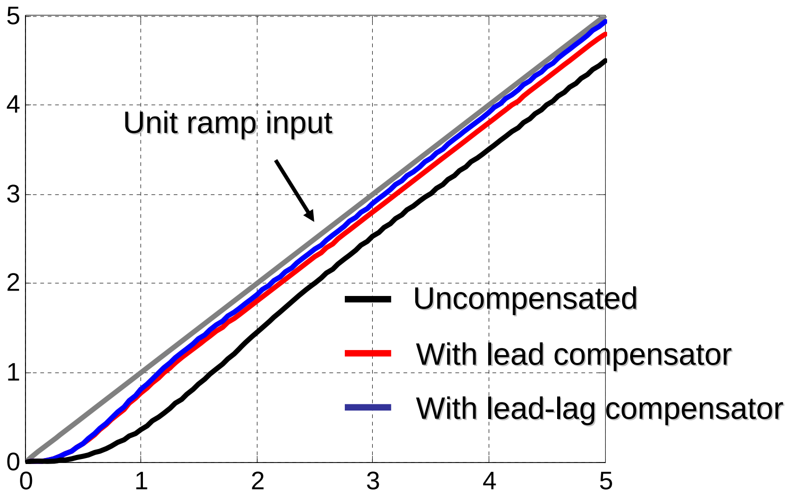
With lead-lag compensator

Desired pole

Comparison of step responses



Comparison of ramp responses



Summary and exercises

- Controller design based on root locus
  - Lag compensator design
    - **Lag compensator improves steady state error.**
  - Lead-lag compensator design
    - **Lead-lag compensator improves stability, transient and steady-state responses.**
- Next, frequency response and Bode plot