ME451: Control Systems

Lecture 19

Root locus: Multiple parameter design

Dr. Jongeun Choi
Department of Mechanical Engineering
Michigan State University

Course roadmap

Modeling
- ✓ Laplace transform
- ✓ Transfer function
- ✓ Models for systems
  - ✓ electrical
  - ✓ mechanical
  - ✓ electromechanical
- ✓ Block diagrams
- ✓ Linearization

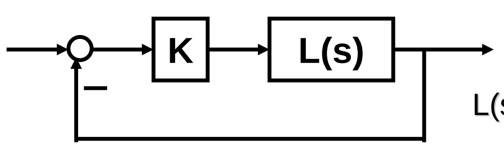
Analysis
- ✓ Time response
  - ✓ Transient
  - ✓ Steady state
- Frequency response
  - Bode plot
- ✓ Stability
  - ✓ Routh-Hurwitz
  - Nyquist

Design
- Design specs
- Root locus
- Frequency domain
- PID & Lead-lag
- Design examples

(Matlab simulations &) laboratories

What is Root Locus? (Review)

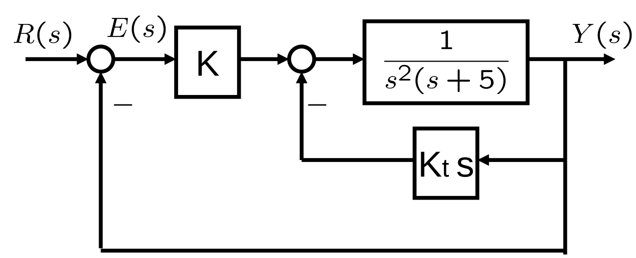
- Consider a feedback system that has one parameter (gain) K>0 to be designed.



L(s): open-loop TF

- *Root locus* graphically shows how poles of CL system varies as K varies from 0 to infinity.
- Today, multiple design parameters!

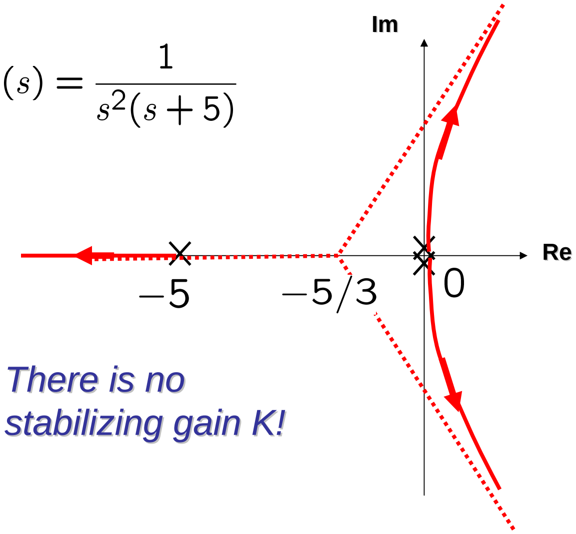
Example 1



a) Set K_t=0. Draw root locus for K>0.
b) Set K=10. Draw root locus for K_t>0.
c) Set K=5. Draw root locus for K_t>0.

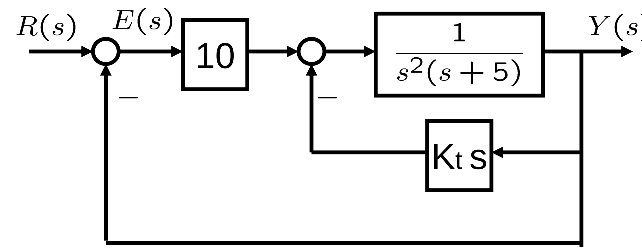
Example 1 (a): Kt=0

$$L(s) = \frac{1}{s^2(s+5)}$$



There is no stabilizing gain K!

Example 1 (b): K=10

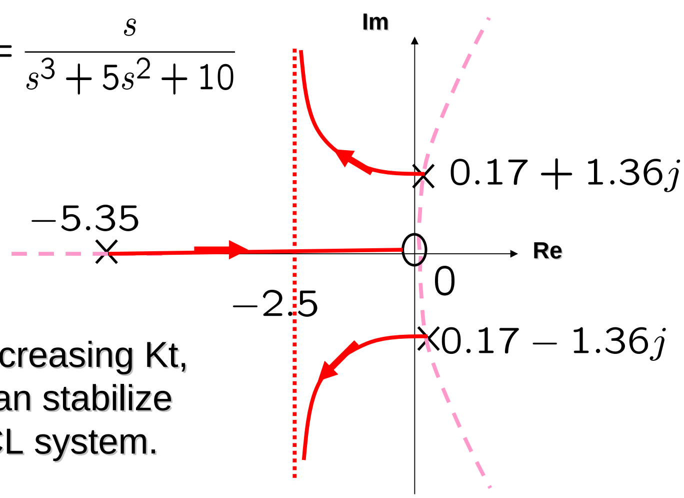


Characteristic eq. $$1 + 10\left(\frac{\frac{1}{s^2(s+5)}}{1+\frac{K_t s}{s^2(s+5)}}\right) = 0$$

$$\Leftrightarrow s^2(s+5) + K_t s + 10 = 0 \Leftrightarrow 1 + K_t \underbrace{\frac{s}{s^3+5s^2+10}}_{L(s)} = 0$$

Example 1 (b)

$$L(s) = \frac{s}{s^3+5s^2+10}$$



By increasing Kt, we can stabilize the CL system.

Finding Kt for marginal stability

- Characteristic equation

$$1+\frac{K_t s}{s^3+5s^2+10} = 0 \Leftrightarrow s^3+5s^2+K_t s+10 = 0$$

- Routh array

s^3	1	K_t
s^2	5	10
s^1	$\frac{5K_t-10}{5}$	
s^0	10	

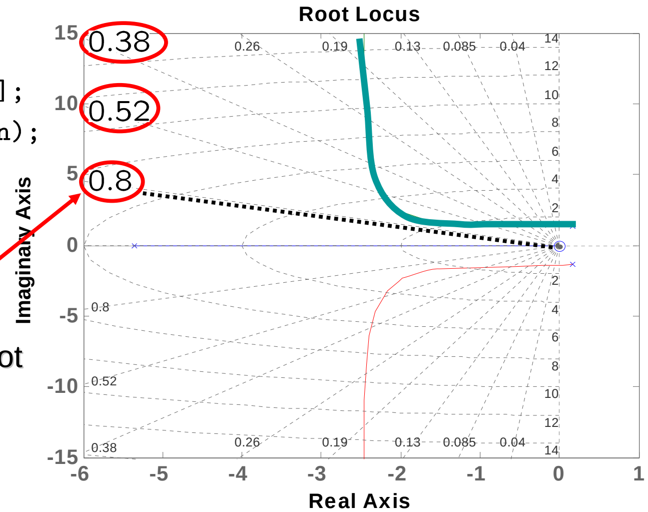
Stability condition $K_t > 2$

- When Kt=2

$$5s^2+10=0 \Rightarrow s = \pm\sqrt{2}j$$

Matlab command “rlocus.m”

```
num=[1 0];
den=[1 5 0 10];
sys=tf(num,den);
rlocus(sys)
grid on
```

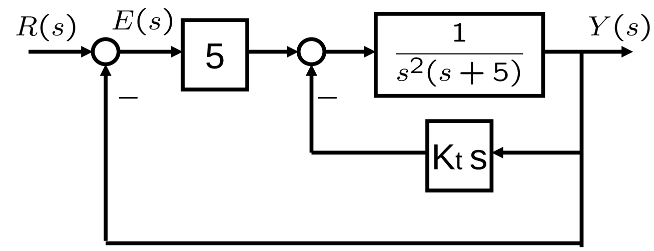


Damping ratio

If K=10, we cannot achieve $\zeta = 0.8$

for any K_t>0.

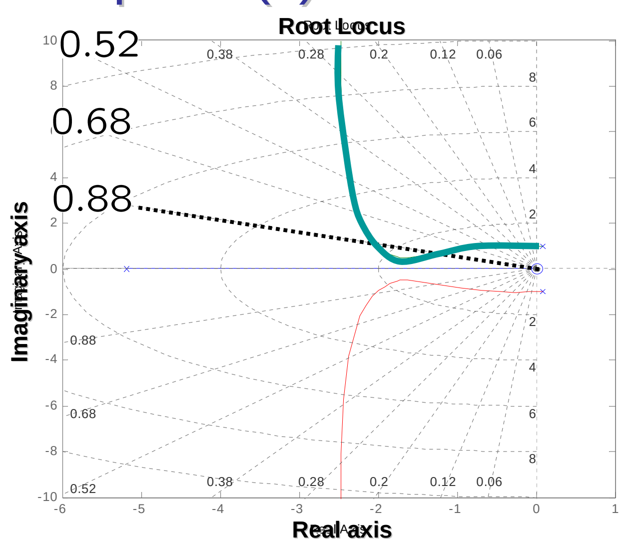
Example 1 (c): K=5



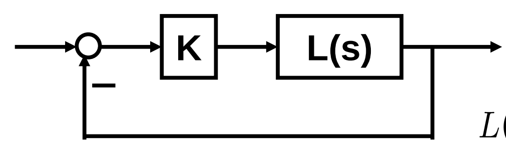
Characteristic eq. $$1+5\left(\frac{\frac{1}{s^2(s+5)}}{1+\frac{K_t s}{s^2(s+5)}}\right)=0$$

$$\Leftrightarrow \; s^2(s+5)+K_t s+5=0 \; \Leftrightarrow \; 1+K_t\underbrace{\frac{s}{s^3+5s^2+5}}_{L(s)}=0$$

Example 1 (c): Root locus plot



Example 2



$$L(s)=\frac{1+Ts}{s(s+1)(s+2)}$$

a) Set T=0. Draw root locus for K>0.

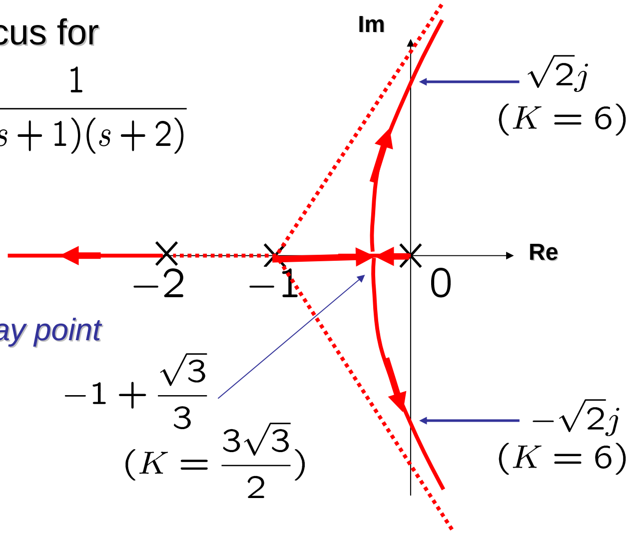
$$L(s)=\frac{1}{s(s+1)(s+2)}$$

b) Vary T to see the effect of a zero on root locus.

Example 2 (a)

- Root locus for

$$L(s) = \frac{1}{s(s+1)(s+2)}$$



Example 2 (b)

- When K is fixed and T is a positive parameter, the characteristic equation can be written as

$$1 + K\frac{1+Ts}{s(s+1)(s+2)} = 0$$

$$\Leftrightarrow \underbrace{s(s+1)(s+2)+K}_{\text{Term without T}} + \underbrace{TKs}_{\text{Term with T}} = 0$$

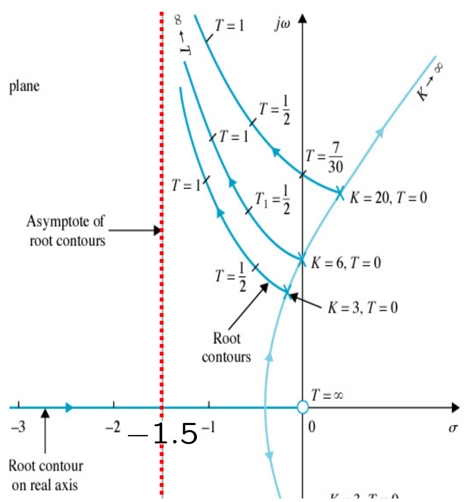
$$\Leftrightarrow 1 + T\frac{Ks}{s(s+1)(s+2)+K} = 0$$

Example 2 (b) (cont'd)

- Root locus for various K & T
- Zero of L(s):

$$s = -\frac{1}{T}$$

- Generally, *addition of a zero improves stability.*



Summary and exercises

- Multiple parameter design examples
- Next, lead compensator design based on root locus
- Exercises
  - For the feedback system,
    - Set a=0, and draw RL for K>0.
    - Set K=9, and draw RL for a>0.