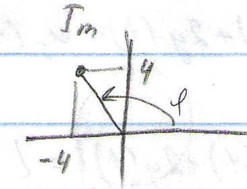
Math Quiz Solutions:

1) $z=(1-j)-(5-5j)=1-5+(-1+5)j=-4+4j$ (1)



$\varphi = 135^\circ$

$|z|=\sqrt{(-4)^2+(4)^2}=5.66$

$\varphi=\arg z=\operatorname{atan2}\left(\frac{4}{-4}\right)=2{,}356$

or $=\operatorname{atan}\left(\frac{4}{-4}\right)+\pi=2{,}356$ $\Rightarrow z=5{,}66\cdot e^{2{,}356 j}$ (1)

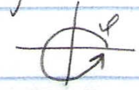
$z=(1+2j)+(2+1j)=1+2+(2+1)j=3+3j$ (1)

$|z|=\sqrt{3^2+3^2}=4{,}24$

$\varphi=\operatorname{atan}\left(\frac{3}{3}\right)=0{,}785=\frac{\pi}{4}$

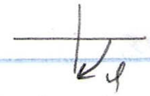
$z=4{,}24\cdot e^{\pi/4 j}=4{,}24\, e^{0{,}785 j}$ (1)

$z=e^{\pi j}e^{\pi j}=e^{2\pi j}$ (1) $=1+0j$ (1)

$\varphi=2\pi \Rightarrow$  $\Rightarrow y=0$

$|z|=1 \Rightarrow \sqrt{x^2}=1 \Rightarrow x=1$

$z=10\dfrac{e^{\pi/2 j}}{e^{\pi j}}=10\, e^{(\frac{\pi}{2}-\pi)j}=10\, e^{-\pi/2 j}$ (1)

$\varphi=-\frac{\pi}{2} \Rightarrow$  $\Rightarrow x=0 \Rightarrow z=0-10j$ (1) Total: 8

2) $\log_{10} x^5 y^{-6}=\log_{10} x^5+\log_{10} y^{-6}=5\log_{10} x-6\log_{10} y$ (2)

$\log_{10}\dfrac{x^{25}}{y^{50}}=\log_{10} x^{25}-\log_{10} y^{50}=25\log_{10} x-50\log_{10} y$ (2) Total: 4

3) $\dfrac{1}{x}=\dfrac{1}{a}+\dfrac{1}{b}=\dfrac{b+a}{ab}=\dfrac{a+b}{ab} \Rightarrow x=\dfrac{ab}{a+b}$ (4)

(-1) Will be for the not simplified result

4) $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $A^{-1}=\dfrac{1}{\det(A)}A^*$ where A^* is adjugate matrix

A^* for the case of dimension of $A=2\times 2$ equals to $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ (1)

$\det A=ad-cb$ (1)

Finally $A^{-1}=\dfrac{1}{ad-cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. (2)

Total: 4

5) $\ddot{y}(t)+3\dot{y}(t)+2y(t)=u_s(t)$ $\quad y(0)=0 \quad \mathcal{L}[u_s(t)]=\frac{1}{s}$

$\dot{y}(0)=0 \quad \mathcal{L}[e^{-\alpha t}]=\frac{1}{s+\alpha}$

$$\mathcal{L}[\ddot{y}(t)+3\dot{y}(t)+2y(t)]=\mathcal{L}[u_s(t)]$$

$$\Rightarrow \underbrace{s^2y-sy(0)-\dot{y}(0)}_{\mathcal{L}[\ddot{y}(t)]}+\underbrace{3[sy-y(0)]}_{\mathcal{L}[\dot{y}(t)]}+\underbrace{2y}_{\mathcal{L}[y(t)]}=\frac{1}{s}$$

use initial conditions:

$$\Rightarrow s^2y-s\cdot1-1+3sy-3\cdot1+2y=\frac{1}{s}$$

group by y

$$[s^2+3s+2]y-4-s=\frac{1}{s}$$

$$[s^2+3s+2]y=\frac{1}{s}+s+4$$

$$[s^2+3s+2]y=\frac{1+s^2+4s}{s} \quad \Rightarrow y(s)=\frac{s^2+4s+1}{s(s^2+3s+2)}=\frac{s^2+4s+1}{s(s+1)(s+2)}$$

$$s^2+3s+2=(s+1)(s+2)$$

$$\Rightarrow y(s)=\frac{A}{s}+\frac{B}{s+1}+\frac{C}{s+2}$$

A, B, C can be founded by different ways.

One of them is the following [find a limit of y(s) with eliminated term which contains zero of function

$$A=\lim_{s\to0} s\cdot y(s)=\frac{0^2+4\cdot0+1}{(0+1)(0+2)}=\frac{1}{2}=0.5$$

$$B=\lim_{s\to-1}(s+1)y(s)=\frac{(-1)^2+4\cdot(-1)+1}{(-1)(-1+2)}=\frac{-2}{-1}=2$$

$$C=\lim_{s\to-2}(s+2)y(s)=\frac{(-2)^2+4(-2)+1}{(-2)(-2+1)}=\frac{-3}{+2}=-\frac{3}{2}=-1.5$$

plug A, B, C into y(s)

$$y(s)=\frac{0.5}{s}+\frac{2}{s+1}-\frac{1.5}{s+2}$$

to find y(t) need to find $\mathcal{L}^{-1}[y(s)]=\mathcal{L}^{-1}\left[\frac{0.5}{s}\right]+\mathcal{L}^{-1}\left[\frac{2}{s+1}\right]-\mathcal{L}^{-1}\left[\frac{1.5}{s+2}\right]$

$$y(t) \quad = 0.5\cdot u_s(t)+2e^{-1t}-1.5e^{-2t}$$ where $u_s(t)$ is a step function

Finally: $y(t)=0.5\,u_s(t)+2e^{-t}-1.5e^{-2t}$

All - OK 8/8

right idea, but wrong calculations (arithmetic) → 4/8

mistake in the approach of getting y(s) with whole other part good in method of solving problem → 2/8 ÷ 3/8

Total: 8.

also if y(s) is right but all later calculations wrong