# ME451: Control Systems

**Lecture 5**
**Modeling of mechanical systems**

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# Course roadmap

***Modeling***

- Laplace transform
- Transfer function
- Models for systems
  - electrical
  - mechanical
  - electromechanical
- Block diagrams
- Linearization

***Analysis***

- Time response
  - Transient
  - Steady state
- Frequency response
  - Bode plot
- Stability
  - Routh-Hurwitz
  - Nyquist

***Design***

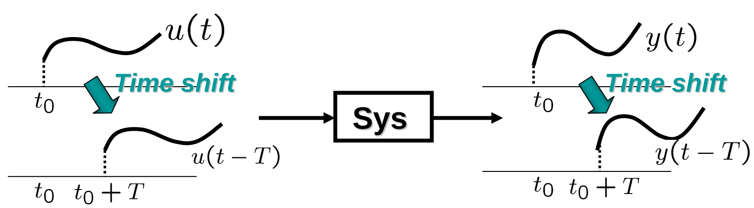
- Design specs
- Root locus
- Frequency domain
- PID & Lead-lag
- Design examples

*(Matlab simulations &) laboratories*



# Time-invariant & time-varying

- A system is called ***time-invariant (time-varying)*** if system parameters do not (do) change in time.
- Example: Mx''(t)=f(t) & M(t)x''(t)=f(t)
- For time-invariant systems:



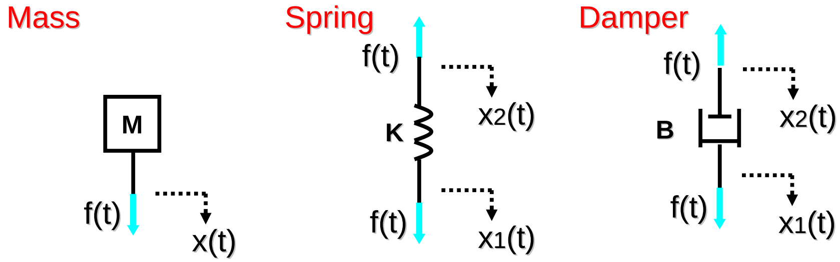

- This course deals with time-invariant systems.



# Newton's laws of motion

- 1st law:
  - A particle remains at rest or continues to move in a straight line with a constant velocity if there is no unbalancing force acting on it.
- 2nd law:
  - $\sum F_i(t) = m\frac{d^2x}{dt^2}$ : translational
  - $\sum \tau_i(t) = I\frac{d^2\theta}{dt^2}$ : rotational
- 3rd law:
  - For every action has an equal and opposite reaction

# Translational mechanical elements: (constitutive equations)



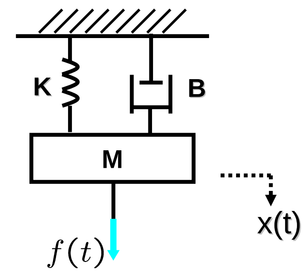

$$f(t) = Mx''(t) \qquad f(t) = K(x_1(t) - x_2(t)) \qquad f(t) = B(x_1'(t) - x_2'(t))$$

$$\begin{pmatrix} x(0)=0 \\ \dot{x}(0)=0 \end{pmatrix} \qquad \begin{pmatrix} x_1(0)=0 \\ x_2(0)=0 \end{pmatrix}$$

$$F(s) = Ms^2X(s) \qquad F(s) = K(X_1(s) - X_2(s)) \qquad F(s) = Bs(X_1(s) - X_2(s))$$

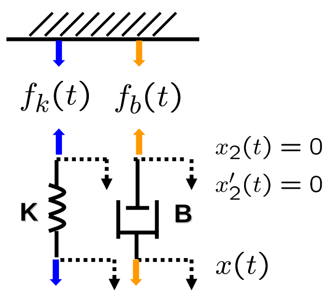


# Mass-spring-damper system

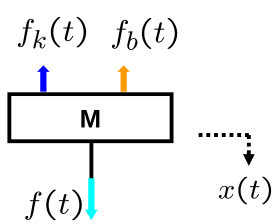





# Free body diagram



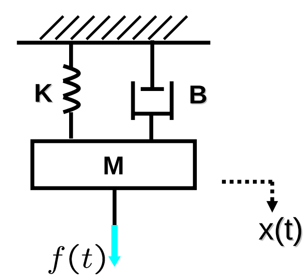

Direction of actual force will be automatically determined by the relative values!




- Newton's law: F=ma



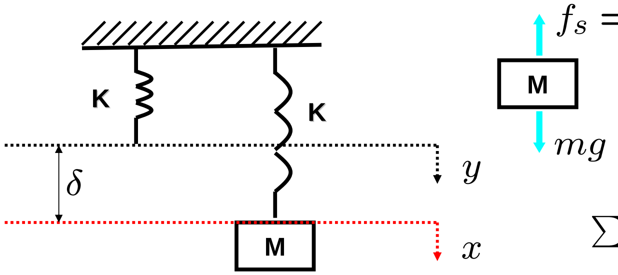
# Mass-spring-damper system




- Equation of motion
- By Laplace transform (with zero initial conditions),

*(2nd order system)*

# Gravity?

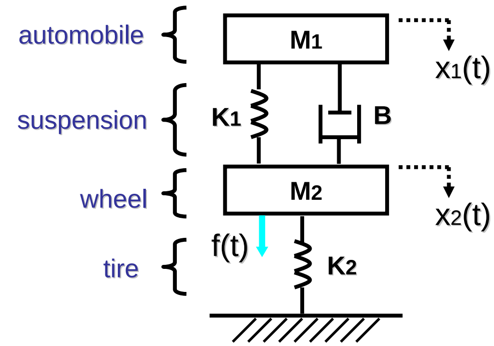



$$\sum F_i = ma$$

- At rest,
- y coordinate:
- x coordinate:



# Automobile suspension system

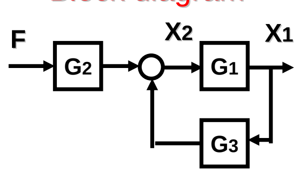





# Automobile suspension system

**Laplace transform with zero ICs**

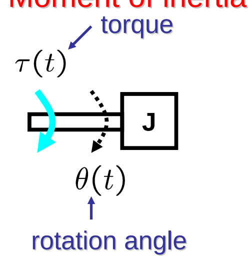
Block diagram



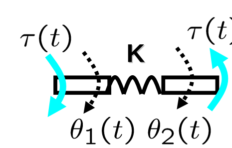



# Rotational mechanical elements (constitutive equations)

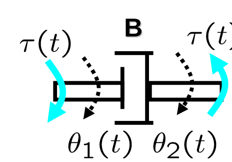
Moment of inertia




Rotational spring




Friction




$$\tau(t) = J\theta''(t)$$

$\begin{pmatrix} \theta(0) = 0 \\ \dot{\theta}(0) = 0 \end{pmatrix}$

$$T(s) = Js^2\Theta(s)$$

$$\tau(t) = K(\theta_1(t) - \theta_2(t))$$

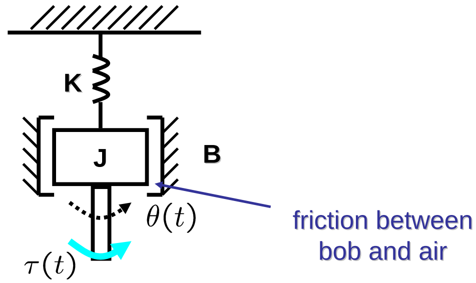
$$T(s) = K(\Theta_1(s) - \Theta_2(s))$$

$$\tau(t) = B(\theta_1'(t) - \theta_2'(t))$$

$\begin{pmatrix} \Theta_1(0) = 0 \\ \Theta_2(0) = 0 \end{pmatrix}$

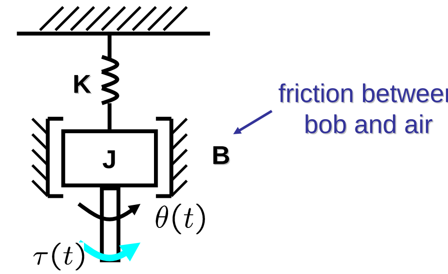
$$T(s) = Bs(\Theta_1(s) - \Theta_2(s))$$

# Torsional pendulum system Ex.2.12






# Torsional pendulum system

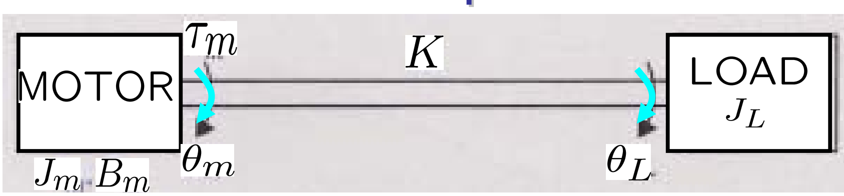



- Equation of Motion
- By Laplace transform (with zero ICs),

*(2nd order system)*



# Example




- By Newton's law
- By Laplace transform (with zero ICs),



# Example (cont'd)

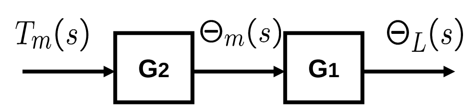
- From second equation:

*(2nd order system)*

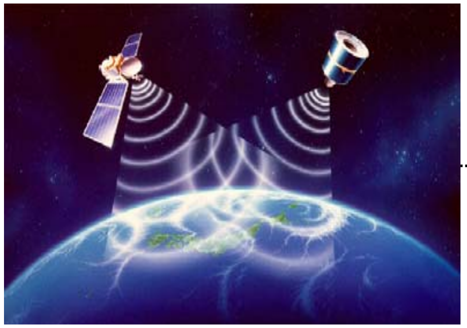
- From first equation:

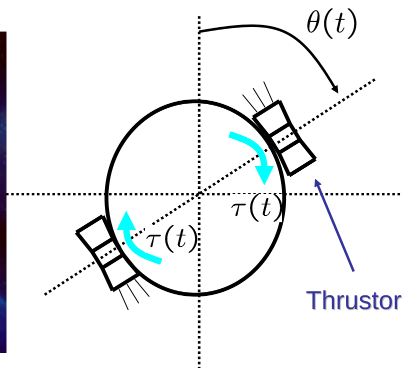
*(4th order system)*

Block diagram



# Rigid satellite Ex. 2.13






- Broadcasting
- Weather forecast
- Communication
- GPS, etc.

$$\tau(t) = J\ddot{\theta}(t)$$

$$\Rightarrow \quad G(s) = \frac{\Theta(s)}{T(s)} = \frac{1}{Js^2}$$

*Double integrator*



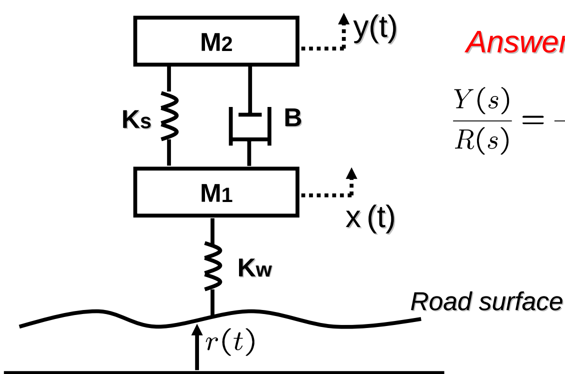
# Summary & Exercises

- Modeling of mechanical systems
  - Translational
  - Rotational
- Next, block diagrams.
- Exercises
  - Read Sections 2.5, 2.6.
  - Derive equations for the automobile suspension problem.



# Exercises (Franklin et al.)

- *Quarter car model*: Obtain a transfer function from R(s) to Y(s).




*Answer*

$$\frac{Y(s)}{R(s)} = \frac{\frac{k_w b}{m_1 m_2}\left(s + \frac{k_s}{b}\right)}{s^4 + \left(\frac{b}{m_1} + \frac{b}{m_2}\right)s^3 + \left(\frac{k_s}{m_1} + \frac{k_s}{m_2} + \frac{k_w}{m_1}\right)s^2 + \left(\frac{k_w b}{m_1 m_2}\right)s + \frac{k_w k_s}{m_1 m_2}}$$