

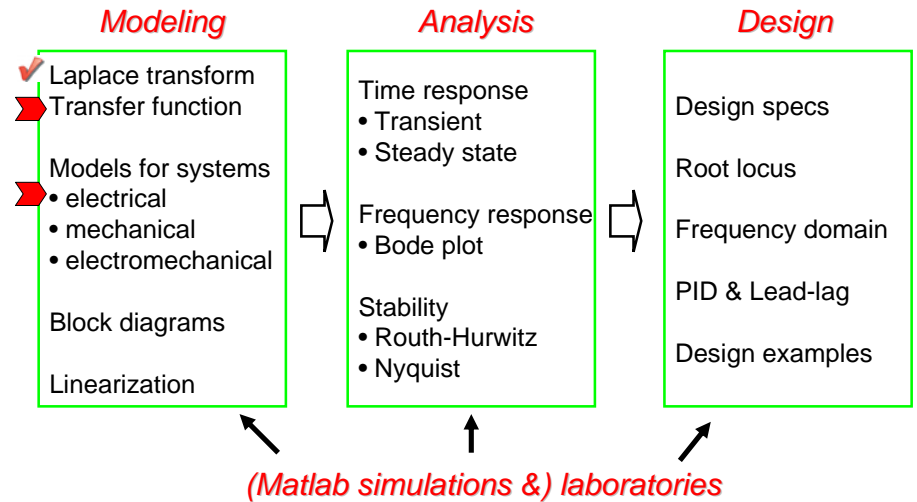
ME451: Control Systems

Lecture 4 Modeling of electrical systems

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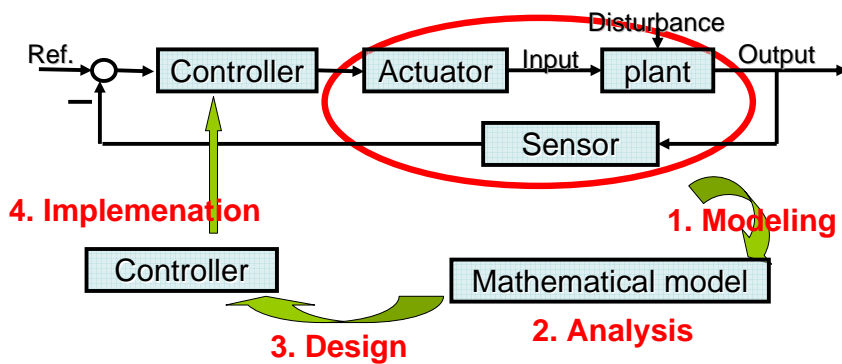
1

Course roadmap



2

Controller design procedure (review)

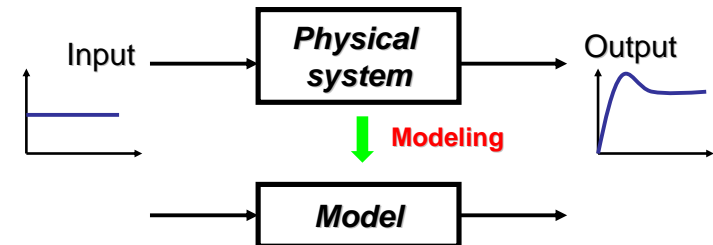


- What is the “mathematical model”?
- Transfer function
- Modeling of electrical circuits

3

Mathematical model

- Representation of the input-output (signal) relation of a physical system



- A model is used for the **analysis** and **design** of control systems.

4

Important remarks on models

- Modeling is the **most important and difficult task** in control system design.
- No mathematical model exactly represents a physical system.

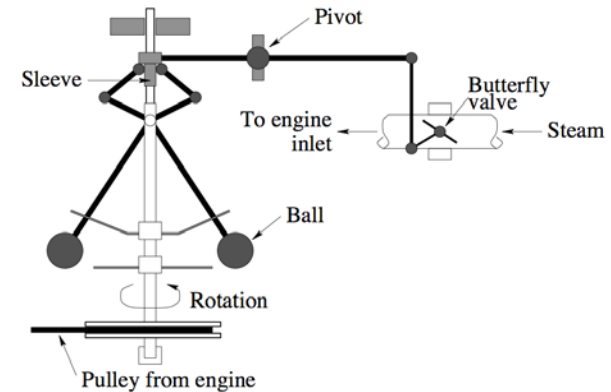
Math model \neq Physical system
 Math model \approx Physical system

- Do not confuse **models** with **physical systems**!
- In this course, we may use the term **“system”** to mean a mathematical model.

5

A Brief Control History

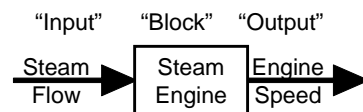
- 1788: James Watt's fly-ball governor
 - Mechanical feedback control of steam supply to an engine



6

The Block Diagram

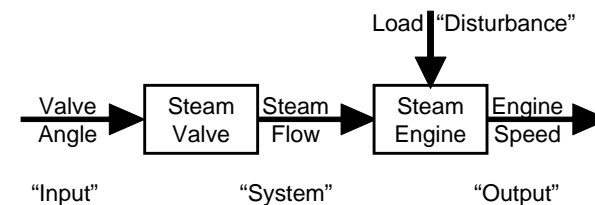
- Communication tool for Engineering Systems
 - Composed of Blocks with inputs and outputs



7

The Block Diagram

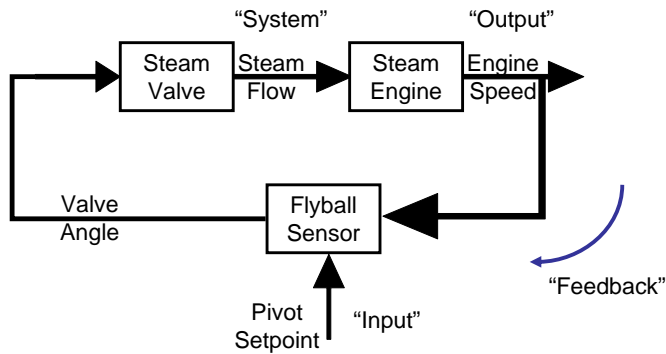
- Blocks Connect to form systems
 - Outputs of one block becomes input to another



8

The Block Diagram

- Blocks Connect to form systems
 - Outputs of one block becomes input to another



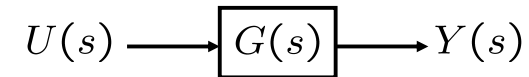
9

Transfer function

- A transfer function is defined by

$$G(s) := \frac{Y(s)}{U(s)}$$

\swarrow Laplace transform of system output
 \swarrow Laplace transform of system input

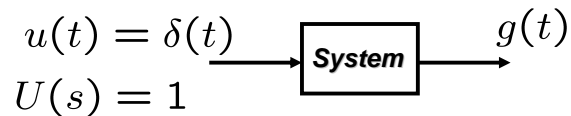


- A system is assumed to be at rest. (Zero initial condition)

10

Impulse response

- Suppose that $u(t)$ is the unit impulse function and system is at rest.

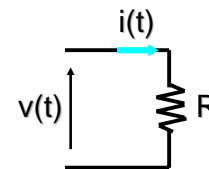


- The output $g(t)$ for the unit impulse input is called **impulse response**.
- Since $U(s)=1$, the transfer function can also be defined as the **Laplace transform of impulse response**: $G(s) := \mathcal{L}\{g(t)\}$

11

Models of electrical elements: (constitutive equations)

Resistance

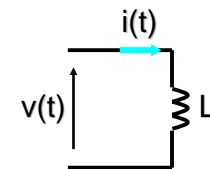


$$v(t) = Ri(t)$$

↓ Laplace transform

$$\frac{V(s)}{I(s)} = R$$

Inductance

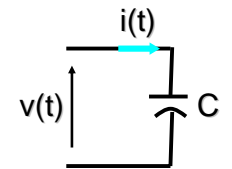


$$v(t) = L \frac{di(t)}{dt}$$

↓ ($i(0) = 0$)

$$\frac{V(s)}{I(s)} = sL$$

Capacitance



$$i(t) = C \frac{dv(t)}{dt}$$

↓ ($v(0) = 0$)

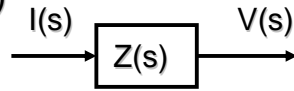
$$\frac{V(s)}{I(s)} = \frac{1}{sC}$$

12

Impedance

- Generalized resistance to a sinusoidal alternating current (AC) $I(s)$

- $Z(s): V(s)=Z(s)I(s)$



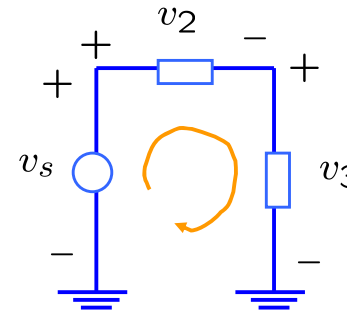
| Element | Time domain | Impedance $Z(s)$ |
|-------------|-----------------------------|------------------------------------|
| Resistance | $v(t) = Ri(t)$ | $\frac{V(s)}{I(s)} = R$ |
| Inductance | $v(t) = L \frac{di(t)}{dt}$ | $\frac{V(s)}{I(s)} = sL$ |
| Capacitance | $i(t) = C \frac{dv(t)}{dt}$ | $\frac{V(s)}{I(s)} = \frac{1}{sC}$ |

Memorize! 13

Kirchhoff's Voltage Law (KVL)

- The algebraic sum of voltage drops around any loop is =0.

$$v_s - v_2 - v_3 = 0$$

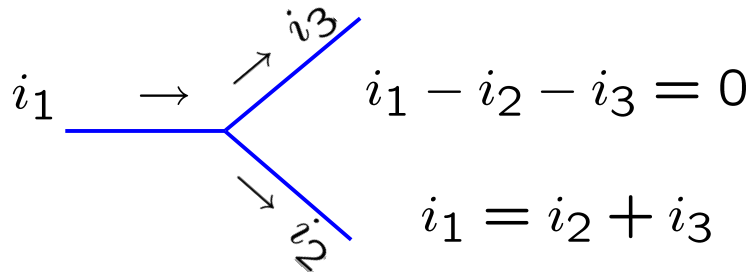


$$v_s = v_2 + v_3$$

14

Kirchhoff's Current Law (KCL)

- The algebraic sum of currents into any junction is zero.

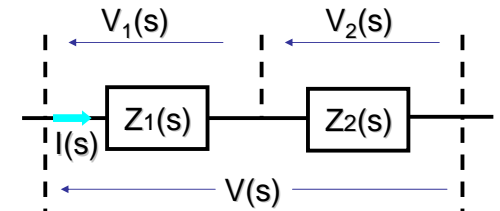


15

Impedance computation

- Series connection

$$Z(s) = Z_1(s) + Z_2(s)$$



- Proof (Ohm's law)

$$V_i(s) = Z_i I(s)$$

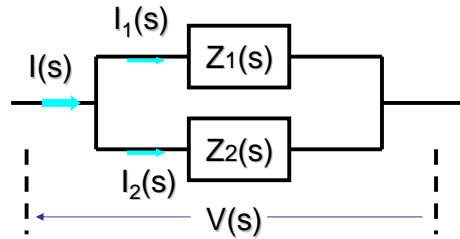
$$V(s) = V_1(s) + V_2(s) = \underbrace{(Z_1(s) + Z_2(s))}_{Z(s)} I(s)$$

16

Impedance computation

- Parallel connection

$$Z(s) = \frac{Z_1(s)Z_2(s)}{Z_1(s) + Z_2(s)}$$



- Proof (Ohm's law)

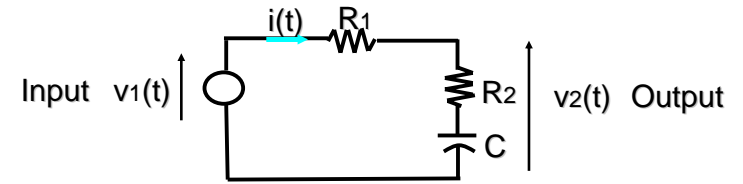
$$V(s) = Z_i I_i(s)$$

- KCL $I(s) = I_1(s) + I_2(s) = \frac{V(s)}{Z_1(s)} + \frac{V(s)}{Z_2(s)}$

$$I(s) = \left(\frac{1}{Z_1(s)} + \frac{1}{Z_2(s)} \right) V(s) = \frac{1}{Z(s)} V(s)$$

17

Modeling example



- Kirchhoff voltage law (with zero initial conditions)

$$v_1(t) = (R_1 + R_2)i(t) + \frac{1}{C} \int_0^t i(\tau) d\tau$$

$$v_2(t) = R_2 i(t) + \frac{1}{C} \int_0^t i(\tau) d\tau$$

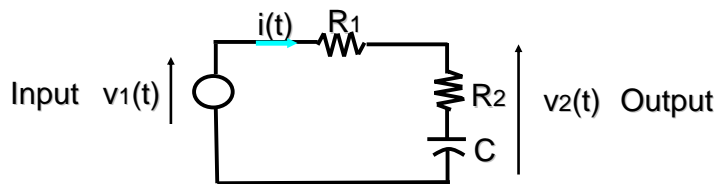
- By Laplace transform,

$$V_1(s) = (R_1 + R_2)I(s) + \frac{1}{sC} I(s)$$

$$V_2(s) = R_2 I(s) + \frac{1}{sC} I(s)$$

18

Modeling example (cont'd)



- Transfer function

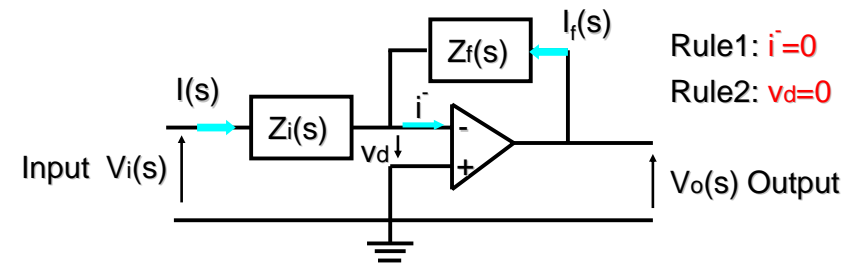
$$G(s) = \frac{V_2(s)}{V_1(s)} = \frac{R_2 + \frac{1}{sC}}{(R_1 + R_2) + \frac{1}{sC}}$$

$$= \frac{R_2 C s + 1}{(R_1 + R_2) C s + 1}$$

(first-order system)

19

Example: Modeling of op amp



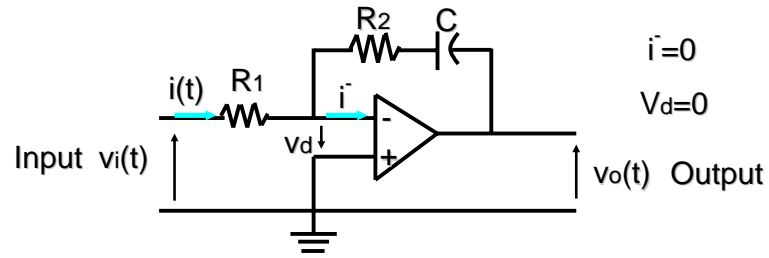
Rule1: $i^- = 0$
Rule2: $v_d = 0$

- Impedance $Z(s)$: $V(s) = Z(s)I(s)$
- Transfer function of the above op amp:

$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_f(s)(I_f(s) = -I(s))}{Z_i(s)I(s)} = -\frac{Z_f(s)}{Z_i(s)}$$

20

Modeling example: op amp



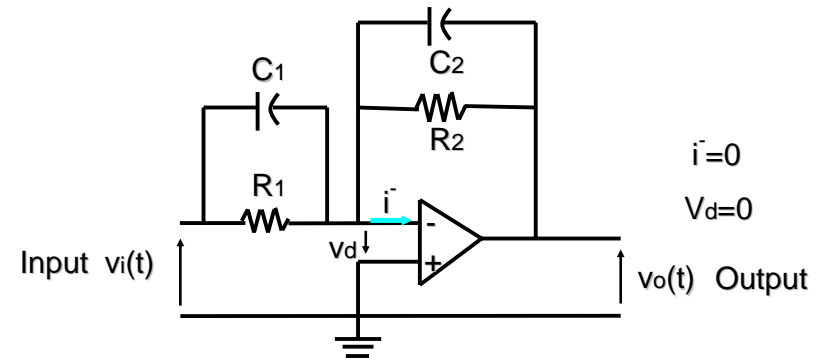
- By the formula in previous two pages,

$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{-(R_2 + \frac{1}{sC})}{R_1} = -\frac{R_2Cs + 1}{R_1Cs}$$

(first-order system)

21

Modeling exercise: op amp

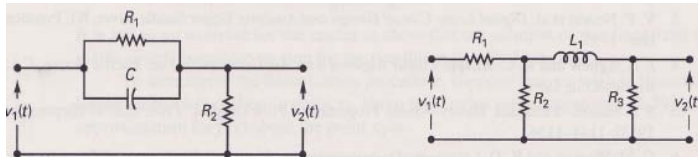


Find the transfer function!

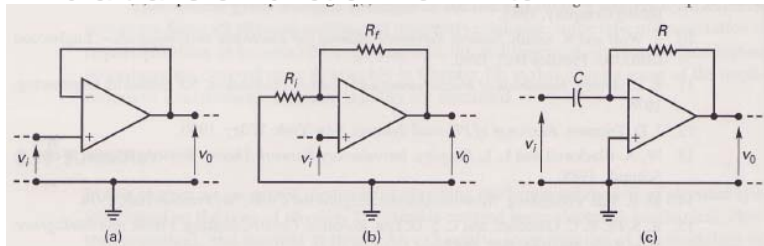
22

More exercises in the textbook

- Find a transfer function from v_1 to v_2 .



- Find a transfer function from v_i to v_o .



23

Summary & Exercises

- Modeling
 - Modeling is an important task!
 - Mathematical model
 - Transfer function
 - Modeling of electrical systems
- Next, modeling of mechanical systems
- Exercises
 - Read Sections 2.2, 2.3
 - Solve problems 2.1, 2.2, and 2.3 in page 62

24