ME451: Control Systems

Lecture 12
Time-domain specifications

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Course roadmap

Modeling
- ✔ Laplace transform
- ✔ Transfer function
- ✔ Models for systems
  - ✔ electrical
  - ✔ mechanical
  - ✔ electromechanical
- ✔ Block diagrams
- ✔ Linearization

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Analysis
- ➤ Time response
  - Transient
  - Steady state
- Frequency response
  - Bode plot
- Stability
  - Routh-Hurwitz
  - Nyquist

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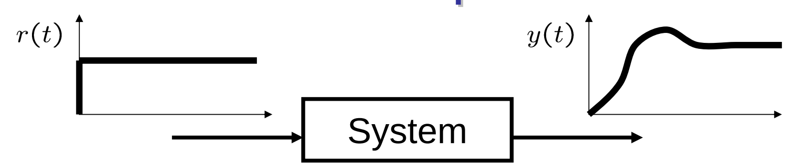
Design
- Design specs
- Root locus
- Frequency domain
- PID & Lead-lag
- Design examples

(Matlab simulations &) laboratories

What we do next

- We will learn stability.
  - Definition in time domain
  - Condition in s-domain
  - Routh-Hurwitz criterion to check the condition
- Stability is a necessary requirement, but not sufficient in most control problems.
- Specifications other than stability
  - How to evaluate a system quantitatively in time domain?
  - How to give specifications in time domain?
  - What are the corresponding conditions in s-domain?

Time response



- We would like to analyze a system property by applying a *test input* r(t) and observing a time response y(t).
- Time response is divided as

$$y(t) = \underline{y_t(t)} + \underline{y_{ss}(t)}$$

Transient response

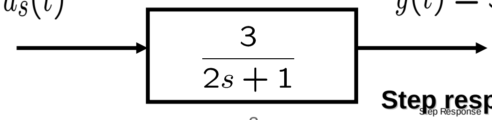
$$\lim_{t\to\infty} y_t(t) = 0$$

Steady-state response

(after y_t dies out)

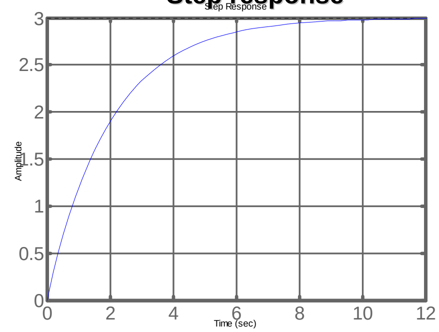
Example of transient & steady-state responses

$r(t) = u_s(t)$ $\quad$ $\dfrac{3}{2s+1}$ $\quad$ $y(t) = 3 - 3e^{-\frac{t}{2}}$



- Transient response
  $y_t(t) = -3e^{-\frac{t}{2}}$
- Steady-state resp.
  $y_{ss}(t) = 3$

Step response

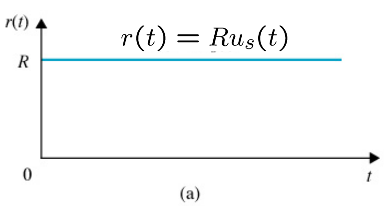


Time (sec)

Usage of time responses

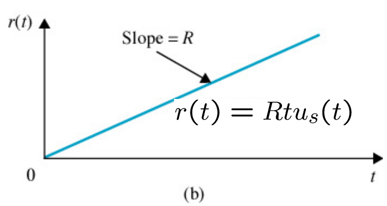
- Modeling
  - Some parameters in the system may be estimated by time responses.
- Analysis
  - Evaluate transient and steady-state responses (Satisfactory or not?)
- Design
  - Given design specs in terms of transient and steady-state responses, design controllers satisfying all the design specs.

Typical test inputs

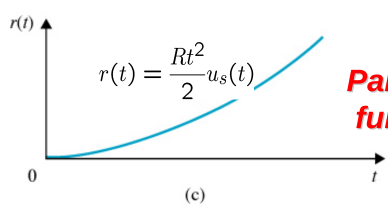


Step function

(Most popular)



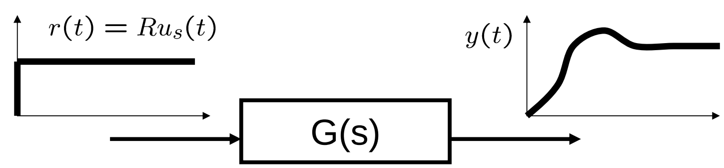
Ramp function



Parabolic function

Sinusoidal input will be dealt with later.

Steady state value for step test signal

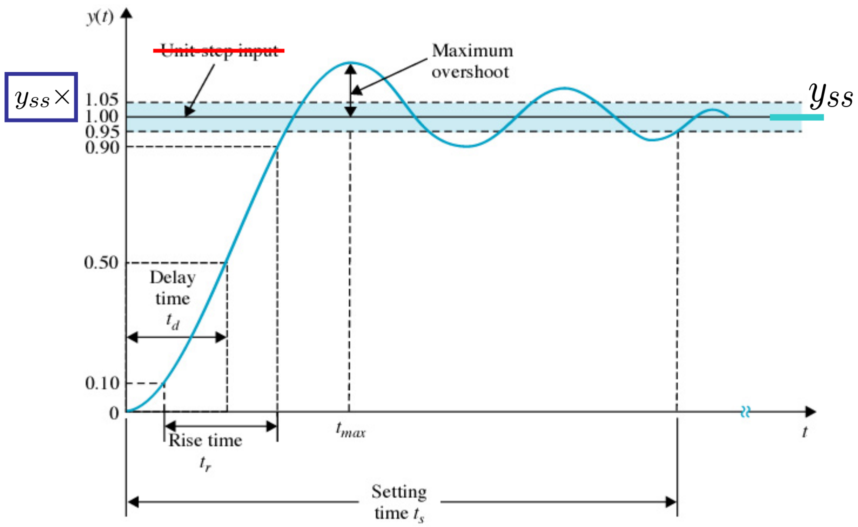


- Suppose that G(s) is stable.
- By the final value theorem:

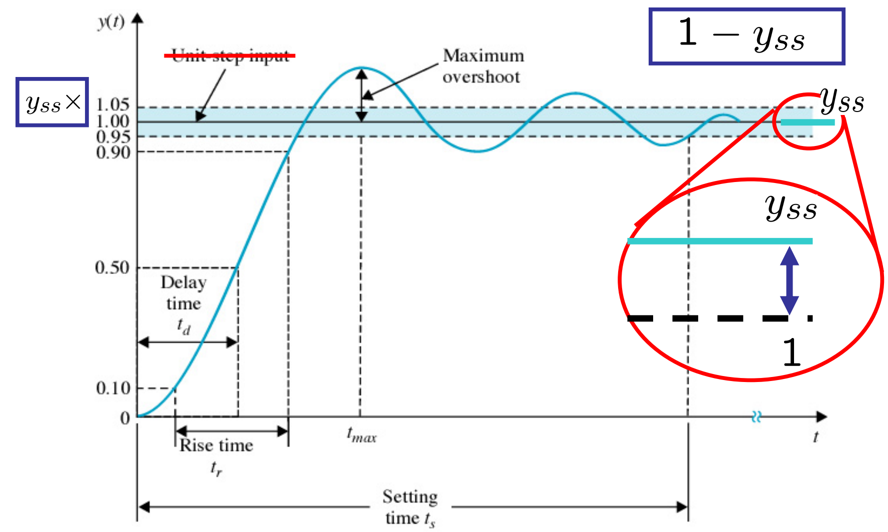
$$y(t) = \lim_{s \to 0} sG(s)\frac{R}{s} = RG(0)$$

- Step response converges to some finite value, called ***steady state value*** y_{ss}.

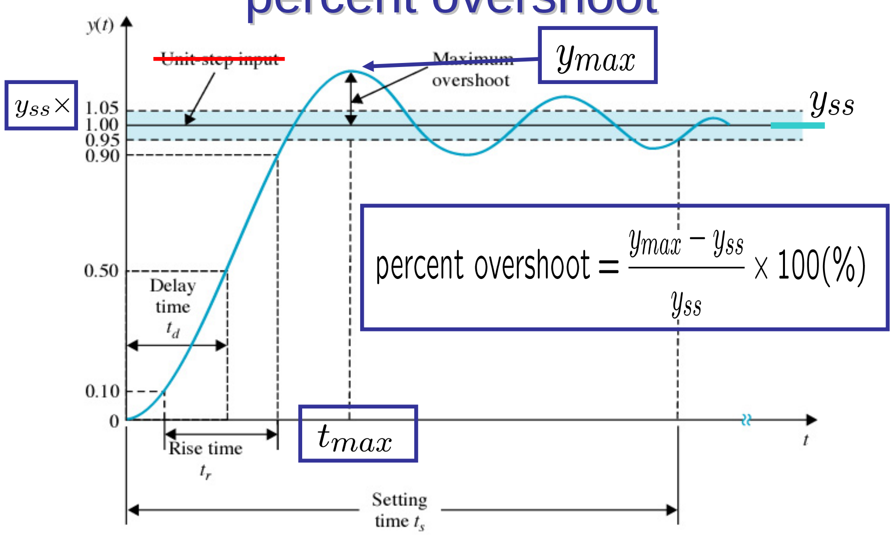
Typical unit step response



Steady-state error for reference us(t)

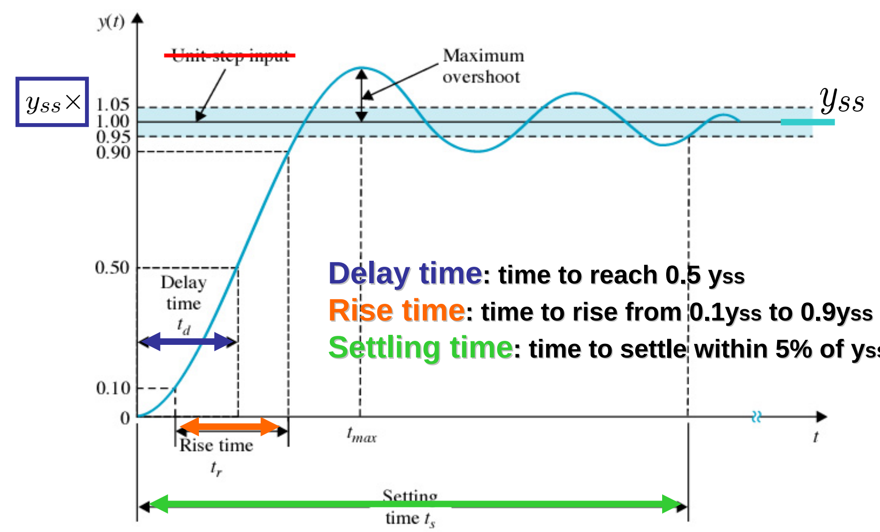


Peak value, peak time, and percent overshoot



$$\text{percent overshoot} = \frac{y_{max} - y_{ss}}{y_{ss}} \times 100(\%)$$

Delay, rise, and settling times



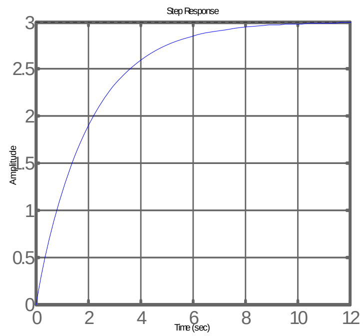
Delay time: time to reach 0.5 y_{ss}

Rise time: time to rise from 0.1y_{ss} to 0.9y_{ss}

Settling time: time to settle within 5% of y_{ss}

An example revisited

- For the example in a previous slide,
  - Steady-state error : 2
  - Delay time around 1.5 sec
  - Rise time around 5 sec
  - Settling time around 6 sec



Remark: There is no peak in this case, so peak value, peak time and percent overshoot cannot be defined.

Remarks on time-domain responses

- Speed of response is measured by
  - Rise time, delay time, and settling time
- Relative stability is measured by
  - Percent overshoot
- In general ….
  - Fast response → Large percent overshoot
  - Large percent overshoot → small stability margin
- We need to take trade-off between response speed and stability.

Summary and Exercises

- Time response and time domain specifications
  - Time response can be used for
    - Parameter estimation
    - Design specification of the feedback system
  - Time response is difficult to compute analytically, except 1st and 2nd order systems (we'll study later).
- Next
  - When does steady state error become zero?
- Exercises
  - Read Section 4.3.