MICHIGAN STATE UNIVERSITY
Department of Mechanical Engineering
ME451 Control Systems Spring 2010

Midterm Exam I

Closed Book. One 8.5×11 page of handwritten note allowed.

Your Name: Solution
Student Number:

Please start with an easy question and try to answer all questions.

Problem:	1	2	3	Total
Max. Grade:	40	30	~~30~~ 40	100
Grade:				

1 Problem

① The Laplace transform of the output signal over the Laplace transform of the input signal

a. (10 points) State two definitions for "transfer function".

② The Laplace transform of the impulse response.

b. (10 points) From Fig. 1, find the transfer function from $u(t)$ to $y(t)$.

$$\boxed{\frac{KG}{1+KG}}$$

c. (20 points) Define the error $e(t)$ by $e(t) = u(t) - y(t)$. Consider a unit step function input $u(t) = 1$, a control gain K and a first-order system $G(s) = \frac{1}{s+1}$. Find a value for K that gives you a steady state error of 0.1, i.e., $e_{ss} := \lim_{t\to\infty} e(t) = 0.1$.

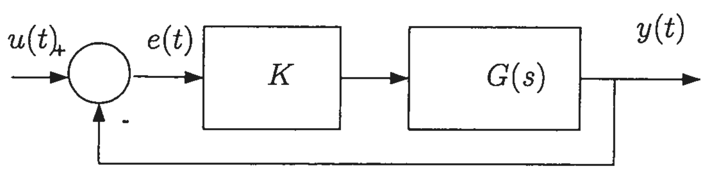


Figure 1: A block diagam.

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s\,E(s) = \lim_{s\to 0} \cancel{s} \cdot \frac{1}{1+K\left(\frac{1}{s+1}\right)} \cdot \frac{1}{\cancel{s}}$$

$$= \frac{1}{1+K} = \frac{1}{10}$$

$$\therefore \boxed{K = 9}$$

2 Problem

Given the differential equation $\dot{x} = f(x, u)$, linearize the equation about $x_0 = 2$ if the function $f(x, u)$ is given by

a. **(10 points)** $f(x, u) = -2x + 2u$,

b. **(10 points)** $f(x, u) = 2^3 - x^3 e^u$.

c. **(10 points)** By comparing the original equations in a. and b. and the resulting linearized equations, explain why you obtain such linearized equations.

(a.) $x_0 = 2 \qquad 0 = -2x_0 + 2u_0 \Rightarrow \quad u_0 = 2$

$$\left.\frac{\partial f}{\partial x}\right|_{x_0, u_0} = -2 \qquad \left.\frac{\partial f}{\partial u}\right|_{x_0, u_0} = 2$$

$x = x_0 + \delta x$

$u = u_0 + \delta u$

$$\boxed{\delta\dot{x} = -2\delta x + 2\delta u}$$

(b.) $x_0 = 2, \qquad 0 = 2^3 - 2^3 e^{u_0} \Rightarrow \quad u_0 = 0$

$$\left.\frac{\partial f}{\partial x}\right|_{x_0 u_0} = -3x^2 e^u = -3 \cdot 4$$

$x = x_0 + \delta x$

$u = u_0 + \delta u$

$$\boxed{\delta\dot{x} = -12\,\delta x - 8\,\delta u}$$

$$\left.\frac{\partial f}{\partial u}\right|_{x_0 u_0} = -x^3 e^u = -8$$

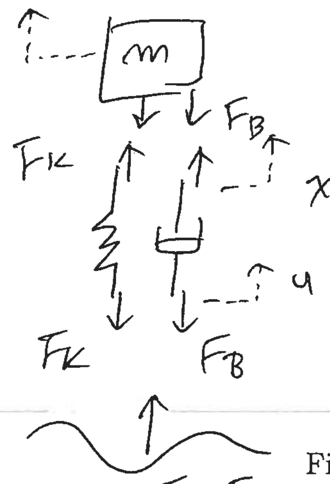
(c) For a. the original equation is already linear. So the linearized eq. @ (x_0, u_0) is the mere ~~to~~ translated equation. They looks the same.

For b, the original equation is nonlinear. So the linearized eq looks different from the original one.

(a)

3 Problem

FBD



$F_K = k(x-u)$

$F_B = b(\dot{x}-\dot{u})$

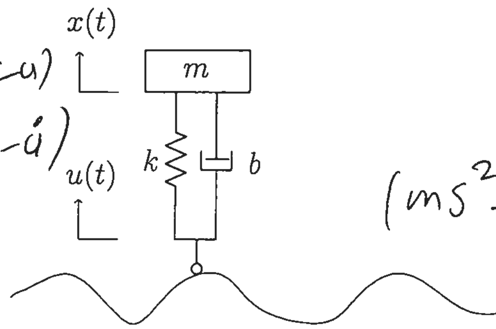


Figure 2: A simplified quarter car model.

(b) $\Sigma F_i = ma$

$$m\ddot{x} = -k(x-u) - b(\dot{x}-\dot{u})$$

$$m\ddot{x} + b\dot{x} + kx = b\dot{u} + kx$$

$\mathcal{L}$

$$(ms^2+bs+k)X(s) = (bs+k)U$$

Figure 2 shows a simplified quarter car model. Assume no gravity. Notations are as follows.

- $u(t)$: the end position of the system with the coordinate shown in Figure 2.
- $x(t)$: the position of the mass with the coordinate shown in Figure 2.
- m : the mass of the car.
- k : the spring constant of a linear spring.
- b : the damping coefficient of a linear damper.

a. **(20 points)** Draw the complete free body diagram of the quarter car system.

b. **(10 points)** Determine the equation of motion.

c. **(10 points)** Determine the transfer function from the input $u(t)$ to the output $x(t)$, i.e.,

$$G(s) := \frac{X(s)}{U(s)} = \frac{\mathcal{L}(x(t))}{\mathcal{L}(u(t))}. \quad = \quad \frac{bs+k}{ms^2+bs+k}$$