ME451: Control Systems

Lecture 6
Block diagrams

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Course roadmap

Modeling	*Analysis*	*Design*
✔ Laplace transform ✔ Transfer function ✔ Models for systems ✔ electrical ✔ mechanical • electromechanical Block diagrams Linearization	Time response • Transient • Steady state Frequency response • Bode plot Stability • Routh-Hurwitz • Nyquist	Design specs Root locus Frequency domain PID & Lead-lag Design examples

(Matlab simulations &) laboratories

Transfer function (review)

- A transfer function is defined by

$$G(s) := \frac{Y(s)}{U(s)}$$

Laplace transform of system output

Laplace transform of system input

$$U(s) \longrightarrow \boxed{G(s)} \longrightarrow Y(s)$$

- A system is assumed to be at rest. (Zero initial condition)

Impulse response (review)

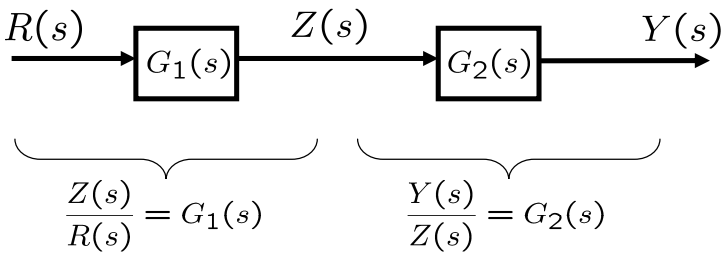
- Suppose that u(t) is the unit impulse function and system is at rest.

$$u(t) = \delta(t) \qquad U(s) = 1 \longrightarrow \boxed{\textbf{System}} \longrightarrow g(t)$$

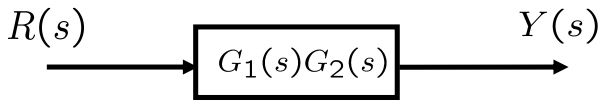
- The output g(t) for the unit impulse input is called ***impulse response***.
- Since U(s)=1, the transfer function can also be defined as the Laplace transform of impulse response: $G(s) := \mathcal{L}\{g(t)\}$

Elementary TF block diagrams

- Series connection

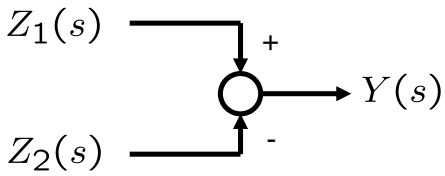


$$\frac{Y(s)}{R(s)} = G_1(s)G_2(s)$$



Elementary TF block diagrams

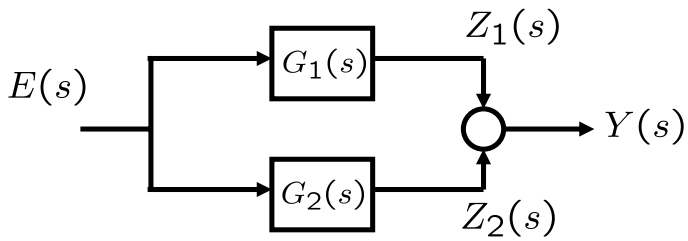
- Summing Junction



$$Y(s) = Z_1(s) - Z_2(s)$$

Elementary TF block diagrams

- Parallel connection

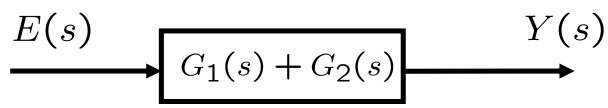


$$\frac{Z_1(s)}{E(s)} = G_1(s)$$

$$\frac{Z_2(s)}{E(s)} = G_2(s)$$

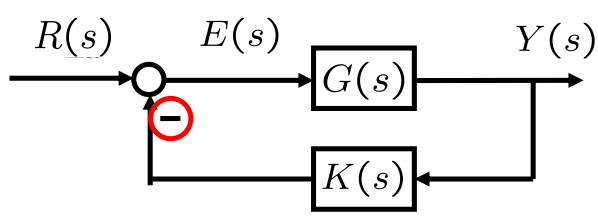
$$Y(s) = Z_1(s) + Z_2(s) = (G_1(s) + G_2(s))E(s)$$

$$\Rightarrow \quad \frac{Y(s)}{E(s)} = G_1(s) + G_2(s)$$



Feedback connection

- Negative feedback system

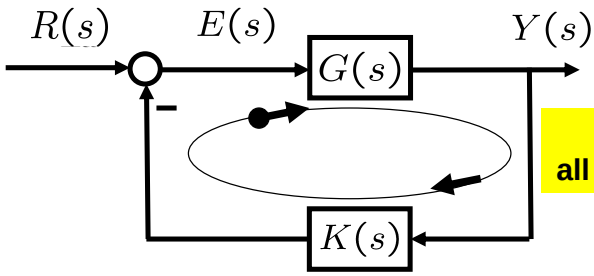


Be careful when computing transfer functions from outside to inside the feedback!

$$E(s) = R(s) - K(s)G(s)E(s) \quad \Rightarrow \quad \frac{E(s)}{R(s)} = \frac{1}{1 + G(s)K(s)}$$

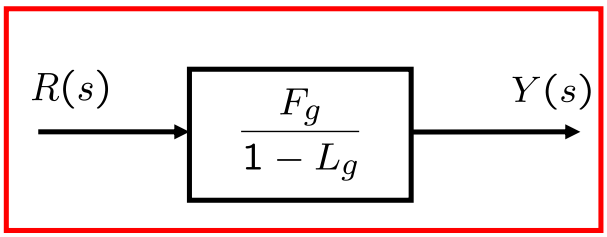
$$Y(s) = G(s)E(s) \quad \Rightarrow \quad \frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)K(s)}$$

Feedback loop formula, $TF_{R \to Y}$



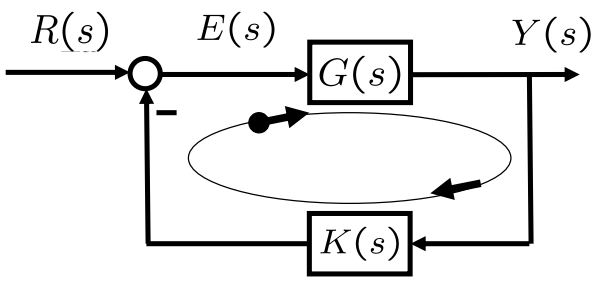
The loop gain is the product of all transfer functions that form the loop

- F_g: Forward gain from R(s) to Y(s) ➡ G(s)
- L_g: Loop gain: ➡ G(s)K(s)(-1)

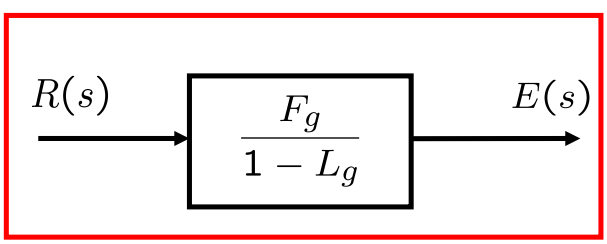


➡ $$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)K(s)}$$

Feedback loop formula, $TF_{R \to E}$

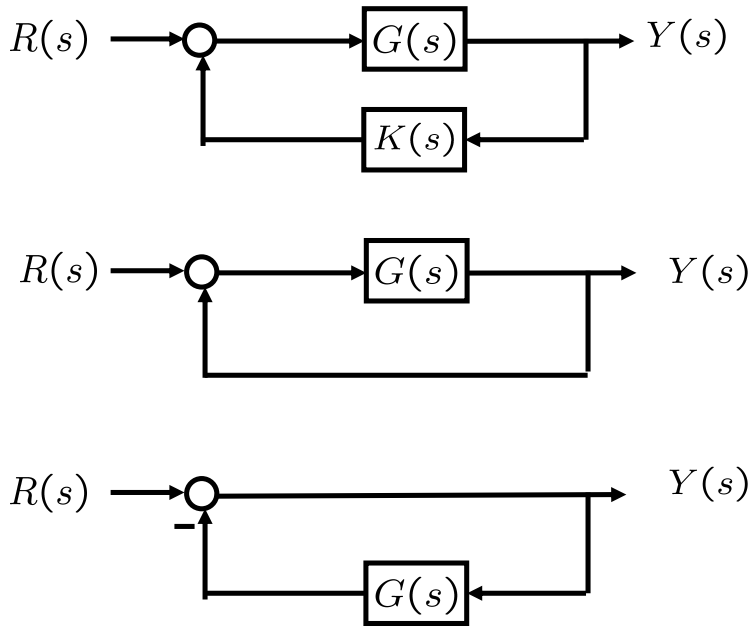


- F_g: Forward gain from R(s) to E(s) ➡ 1
- L_g: Loop gain: ➡ G(s)K(s)(-1)



➡ $$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)K(s)}$$

Exercises



Summary and Exercises

- Block diagrams
  - Elementary diagrams
  - Feedback connections
- Next
  - Linearization
- Exercises
  - Obtain TFs from R to Y on the previous page.
  - Obtain TFs from input F to outputs x_1 and x_2 for the quarter car problem in terms of G_1, G_2, and G_3.