ME451: Control Systems

Lecture 6
Block diagrams

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Course roadmap

Modeling
- Laplace transform
- Transfer function
- Models for systems
  - electrical
  - mechanical
  - electromechanical
- Block diagrams
- Linearization

Analysis
- Time response
  - Transient
  - Steady state
- Frequency response
  - Bode plot
- Stability
  - Routh-Hurwitz
  - Nyquist

Design
- Design specs
- Root locus
- Frequency domain
- PID & Lead-lag
- Design examples

(Matlab simulations & laboratories)
Transfer function (review)

- A transfer function is defined by

\[ G(s) := \frac{Y(s)}{U(s)} \]

- A system is assumed to be at rest. (Zero initial condition)

Impulse response (review)

- Suppose that \( u(t) \) is the unit impulse function and system is at rest.

\[ u(t) = \delta(t) \quad \text{System} \quad g(t) \]

- The output \( g(t) \) for the unit impulse input is called impulse response.

- Since \( U(s)=1 \), the transfer function can also be defined as the Laplace transform of impulse response:

\[ G(s) := \mathcal{L} \{g(t)\} \]
Elementary TF block diagrams

- Series connection

\[
\begin{align*}
R(s) & \rightarrow G_1(s) \rightarrow Z(s) \rightarrow G_2(s) \rightarrow Y(s) \\
\frac{Z(s)}{R(s)} &= G_1(s) \\
\frac{Y(s)}{Z(s)} &= G_2(s) \\
Y(s) &= G_1(s)G_2(s)
\end{align*}
\]

- Summing Junction

\[
\begin{align*}
Z_1(s) & \rightarrow + \rightarrow Y(s) \\
- & \rightarrow Z_2(s) \rightarrow Y(s) \\
Y(s) &= Z_1(s) - Z_2(s)
\end{align*}
\]
Elementary TF block diagrams

- **Parallel connection**

\[ \begin{align*}
E(s) & \quad G_1(s) \quad Z_1(s) \\
& \quad G_2(s) \quad Z_2(s) \\
& \quad Y(s)
\end{align*} \]

\[ \frac{Z_1(s)}{E(s)} = G_1(s) \quad \frac{Z_2(s)}{E(s)} = G_2(s) \]

\[ Y(s) = Z_1(s) + Z_2(s) = (G_1(s) + G_2(s))E(s) \]

\[ \frac{Y(s)}{E(s)} = G_1(s) + G_2(s) \]

- **Feedback connection**

**Negative feedback system**

\[ \begin{align*}
R(s) & \quad E(s) \quad G(s) \quad Y(s) \\
& \quad K(s)
\end{align*} \]

- Be careful when computing transfer functions from outside to inside the feedback!

\[ \frac{E(s)}{R(s)} = \frac{1}{1 + G(s)K(s)} \]

\[ \frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)K(s)} \]
Feedback loop formula, $TF_{R\to Y}$

- $F_g$: Forward gain from $R(s)$ to $Y(s)$  $\Rightarrow$  $G(s)$
- $L_g$: Loop gain:  $\Rightarrow$  $G(s)K(s)(-1)$

\[
\frac{F_g}{1 - L_g}
\]

Feedback loop formula, $TF_{R\to E}$

- $F_g$: Forward gain from $R(s)$ to $E(s)$  $\Rightarrow$  $1$
- $L_g$: Loop gain:  $\Rightarrow$  $G(s)K(s)(-1)$

\[
\frac{1}{1 + G(s)K(s)}
\]
Exercises

\[ R(s) \rightarrow G(s) \rightarrow Y(s) \]

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\[ R(s) \rightarrow G(s) \rightarrow Y(s) \]

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Summary and Exercises

- Block diagrams
  - Elementary diagrams
  - Feedback connections
- Next
  - Linearization
- Exercises
  - Obtain TFs from R to Y on the previous page.
  - Obtain TFs from input F to outputs \( x_1 \) and \( x_2 \) for the quarter car problem in terms of \( G_1, G_2, \) and \( G_3 \).