MICHIGAN STATE UNIVERSITY  
Department of Mechanical Engineering  
ME451 Control Systems Fall 2008  
Midterm Exam II

Closed Book. One 8.5 x 11 page of handwritten note allowed.

Your Name:  

Student Number:

Please start with an easy question and try to answer all questions.

<table>
<thead>
<tr>
<th>Problem</th>
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<tr>
<td>Max. Grade</td>
<td>40</td>
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<td>20</td>
<td>100</td>
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1 Problem

Answer the following questions briefly.

a. (5 points) Write the definition of BIBO (Bounded-Input-Bounded-Output) stability.

b. (5 points) Write the definition of asymptotic stability.

c. (5 points) Consider a transfer function $G(s)$. What is the condition for which $G(s)$ is stable?

d. (15 points) Determine if $G(s)$ in the table is stable, marginally stable, or unstable.

<table>
<thead>
<tr>
<th>$G(s)$</th>
<th>stable/ marginally stable/ unstable</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{(s+1)}{(s+1)(s+0.1)}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{s-1}{s^2+s+1}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{s-1}{(s^2+2)^2}$</td>
<td></td>
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</tbody>
</table>

e. (10 points) What is root locus?
a. Any bounded input generates a bounded output.

b. Any ICs generates y(t) converging to zero.

c. All the poles of G(s) are in the open left half of the complex plane.

d. \[
\frac{s+1}{(s+1)(s+0.1)}\quad \text{stable}
\]

\[
\frac{s-1}{s^2+s+1}\quad \text{stable}
\]

\[
\frac{s-1}{(s^2+2)^2}\quad \text{unstable}
\]

e. Root locus graphically shows how poles of CL system varies as k varies from 0 to infinity.
2 Problem

For the depicted closed-loop system ($\zeta > 0$, $\omega_n > 0$) in Figure 1, answer the following questions.

a. (5 points) Obtain the transfer function from $U(s)$ to $Y(s)$.

b. (5 points) Discuss the stability of the closed-loop system.

c. (5 points) Compute the DC gain of the obtained transfer function.

d. (5 points) Sketch roughly the step response of the closed-loop system, in the case of $0 < \zeta < 1$ (underdamped case).

e. Consider the error signal $e(t) := u(t) - y(t)$. Compute the steady state error for

   i. (5 points) $u(t) = u_s(t)$ (unit step input)
   
   ii. (5 points) $u(t) = tu_s(t)$ (unit ramp input)

f. (10 points) For the system with $\zeta = \frac{1}{\sqrt{2}}$, and $\omega_n = 5$ (rad/sec), obtain percent overshoot and 5% settling time.

![Figure 1: A closed-loop system.](image-url)
\[
\frac{y(t)}{u(t)} = \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} = \frac{\frac{\omega_n^2}{s^2 + 2\zeta \omega_n s}}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\]

b. Since \( \zeta > 0 \) and \( \omega_n > 0 \), the system is stable.

c. DC gain is 1.

d. 
\[
\begin{array}{c}
\text{Graph}
\end{array}
\]

e. 
\[
(i) \quad k_p = \lim_{s \to 0} G(s) = \lim_{s \to 0} \frac{\omega_n^2}{s(s + 2\zeta \omega_n)} = \infty
\]
\[
e_{ss} = \frac{1}{1 + k_p} = 0
\]
\[
(ii) \quad k_v = \lim_{s \to 0} s G(s) = \lim_{s \to 0} s \cdot \frac{\omega_n^2}{s(s + 2\zeta \omega_n)} = \frac{\omega_n}{2\zeta}
\]
\[
e_{ss} = \frac{1}{k_v} = \frac{2\zeta}{\omega_n}
\]

f. 
\[
\text{p.o.} = e^{-\frac{s_1}{\zeta}} \times 100\% = 4.32\%
\]
\[
t_e = \frac{2}{3} \omega_n = 0.85 \text{ sec}
\]
3 Problem

Consider the closed-loop system in Figure 2. The transfer function $G(s)$ is given by

$$G(s) = \frac{K(s + 4)}{s(s + 1)(s + 2)}.$$  

a. (10 points) Using the Routh-Hurwitz criterion, determine the range of $K$ for which the closed-loop system is stable.

b. Consider the error signal $e(t) = r(t) - y(t)$. Compute the steady state error for

i. (5 points) $R(s) = \frac{1}{s}$ (unit step input)

ii. (5 points) $R(s) = \frac{1}{s^2}$ (unit ramp input)

![Figure 2: A closed-loop system.](image)
a. \[ G(s) = \frac{k(s+4)}{s(s+1)(s+2)} \]

Characteristic Equation:
\[ s(s+1)(s+2) + k(s+4) = 0 \]
\[ s^3 + 3s^2 + (2+k)s + 4k = 0 \]

\[ \begin{array}{c|ccc}
 s^3 & 1 & k+2 \\
 s^2 & 3 & 4k \\
 s^1 & \frac{3k+6-4k}{3} & - \frac{k+6}{3} < 0 \Rightarrow k < 6 \\
 s^0 & 4k & 4k > 0 \Rightarrow k > 0 \\
\end{array} \]

\[ \Rightarrow 0 < k < 6 \]

b. (i) \[ k_p = \lim_{{s \to 0}} G(s) = \infty \]
\[ E_{ss} = \frac{1}{1 + k_p} = 0 \]

(ii) \[ k_v = \lim_{{s \to 0}} G(s) = 2k \]
\[ E_{ss} = \frac{1}{k_v} = \frac{1}{2k} \]