Review

~~Transfer Function~~ **Linear Systems**

inputs

outputs

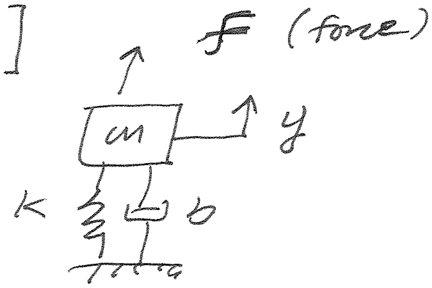
$$\alpha_1 u_1(t) + \alpha_2 u_2(t) \longrightarrow \boxed{H} \longrightarrow \alpha_1 y_1(t) + \alpha_2 y_2(t)$$

where ~~$H[\ldots]$~~ $y_1(t) = H[u_1(t)]$

$y_2(t) = H[u_2(t)]$

Transfer function [input & output map]

$$TF = \frac{\mathcal{L}[\text{output}]}{\mathcal{L}[\text{Input}]} = \mathcal{L}[\text{impulse response}]$$



$$m\ddot{y} = -b\dot{y} - ky + f(t)$$

$\mathcal{L}$

$$(s^2 m + bs + k)Y(s) = F(s)$$

$Y(s) = \mathcal{L}[y(t)]$ ← output

$F(s) = \mathcal{L}[f(t)]$ ← input

$$\underset{(F\to Y)}{TF} = \frac{Y(s)}{F(s)} = \frac{1}{ms^2 + bs + k} = G(s)$$

$$F(s) \longrightarrow \boxed{\frac{1}{ms^2+bs+k}} \longrightarrow Y(s)$$

$\mathcal{L}^{-1}[\mathcal{L}[y_{im}$

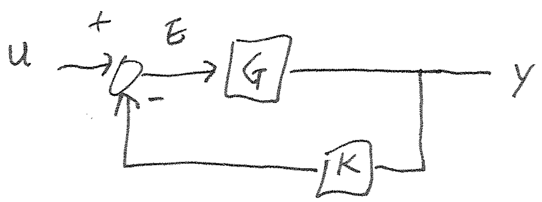
Impulse Response in time domain

$$Y_{im}(s) = G(s)\cdot 1 \xrightarrow{\mathcal{L}^{-1}} y_{im}(t) = \mathcal{L}^{-1}[G(s)]$$

Step – $Y_{step}(s) = G(s)\cdot\frac{1}{s} \xrightarrow{\mathcal{L}^{-1}} y_{step}(t) = \mathcal{L}^{-1}[G(s)\frac{1}{s}]$

Transfer function, Input - output map

I



$$E = u - yK$$

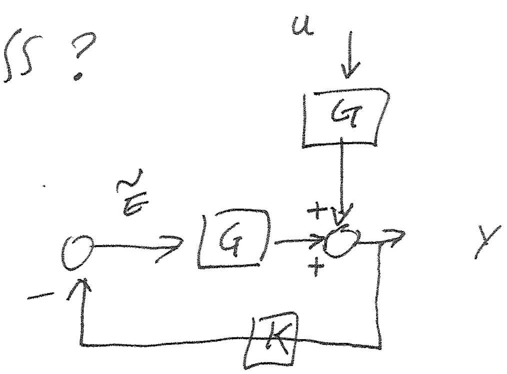
$$Y = EG$$

$$\rightarrow y = G[u - yK]$$

$$Y[1 + KG] = Gu$$

$$\frac{Y}{u} = \frac{G}{1+KG}$$

II SS ?



$$\tilde{E} = -Ky$$

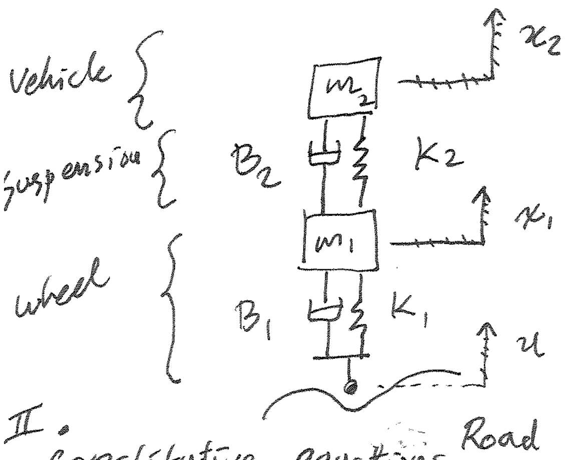
$$Y = \tilde{E}G + uG$$

$$= -KGy + uG$$

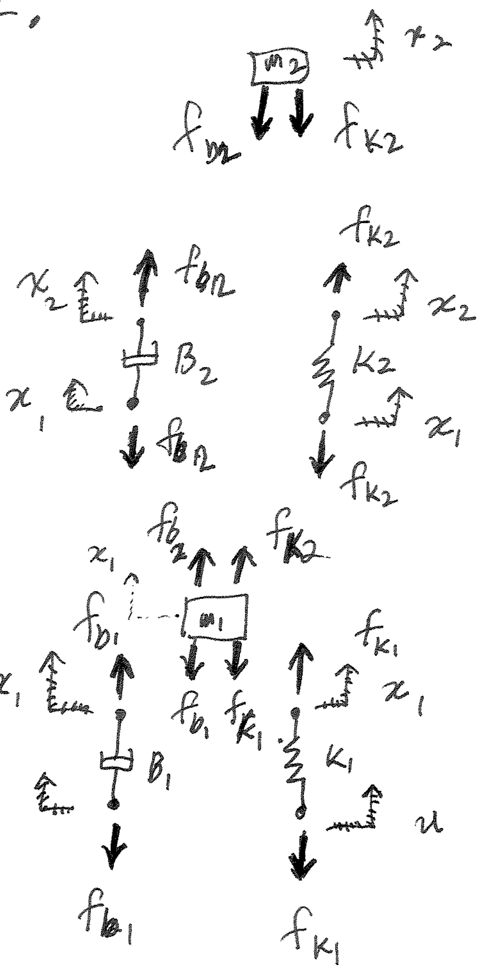
$$Y[1 + KG] = uG$$

$$\frac{Y}{u} = \frac{G}{1+KG}$$

quarter Car Model $\frac{X_2}{X_1}$? TF: $\frac{X_2(s)}{U(s)}$?



I.



II. Constitutive equations

$$f_{b_2} = B_2(\dot{x}_2 - \dot{x}_1)$$

$$f_{K2} = K_2(x_2 - x_1)$$

$$f_{b_1} = B_1(\dot{x}_1 - \dot{u})$$

$$f_{K1} = K_1(x_1 - u)$$

III.

From ① $\Sigma F_i = ma$

$$\Rightarrow \quad m_2\ddot{x}_2 = -f_{b2} - f_{K2} = -B_2(\dot{x}_2 - \dot{x}_1) - K_2(x_2 - x_1)$$

$$m_2\ddot{x}_2 + B_2\dot{x}_2 + K_2x_2 = B_2\dot{x}_1 + K_2x_1$$

$\mathcal{L}$

$$(m_2s^2 + B_2s + K_2)X_2(s) = (B_2s + K_2)X_1(s)$$

$$\text{TF: } \frac{X_2(s)}{X_1(s)} = \underbrace{\frac{(B_2s + K_2)}{m_2s^2 + B_2s + K_2}}_{= G_1(s)}$$

$$X_1(s) \to \boxed{G_1} \to X_2(s)$$

From ② $\Sigma F_i = ma$

$$m_1 \ddot{x}_1 = f_{b_2} + f_{k_2} - f_{b_1} - f_{k_1}$$

$$= B_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) - B_1(\dot{x}_1 - \dot{u}) - k_1(x_1 - u)$$

$$m_1\ddot{x}_1 + (B_1+B_2)\dot{x}_1 + (k_1+k_2)x_1 = B_2\dot{x}_2 + K_2 x_2 + B_1\dot{u} + K_1 u$$

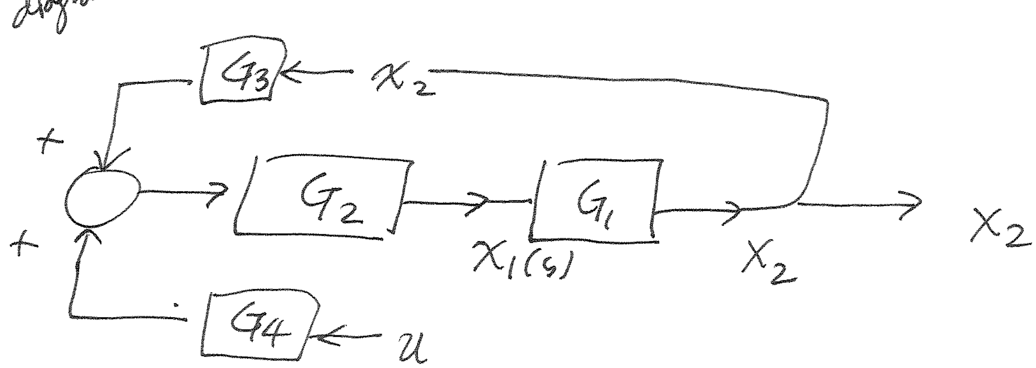
$\downarrow \mathcal{L}$

$$[m_1 s^2 + (B_1+B_2)s + (k_1+k_2)]X_1(s) = (B_2 s + k_2)X_2(s) + (B_1 s + K_1)U(s)$$

$$X_1(s) = \underbrace{\boxed{\frac{1}{m_1s^2+(B_1+B_2)s+(k_1+k_2)}}}_{G_2}\left\{\underbrace{(B_2 s+k_2)}_{G_3} X_2(s) + \underbrace{(B_1 s+K_1)}_{G_4} U(s)\right\}$$

$$X_1(s) = G_2[G_3 X_2 + G_3 u]$$

III. Block diagram



$$\text{TF: } \frac{X_2(s)}{u(s)} = \frac{G_4 G_2 G_1}{1 - G_1 G_2 G_3}$$

$$\frac{\dfrac{(B_2 s+K_2)}{m_2 s^2+B_2 s+K_2}\cdot\dfrac{1}{m_1 s^2+(B_1+B_2)s+(K_1+K_2)}(B_1 s+K_1)}{1-\dfrac{(B_2 s+K_2)}{m_2 s^2+B_2 s+K_2}\cdot\dfrac{1}{m_1 s^2+(B_1+B_2)s+(K_1+K_2)}\cdot(B_2 s+K_2)}$$

$$=\frac{(B_2 s+K_2)(B_1 s+K_1)}{(m_2 s^2+B_2 s+K_2)(m_1 s^2+(B_1+B_2)s+K_1+K_2)-(B_2 s+K_2)^2}$$