prob 0

use 4.4 Response to Harmonic Excitation

or Frequency Response

input $r(t) = \cos \omega t \xrightarrow{\mathcal{L}} R(s) = \frac{s}{s^2+\omega^2}$

output $Y(s) = F_L(s) \cdot R(s) = F_L(s) \cdot \frac{s}{s^2+\omega^2}$ —— ①

partial fraction expansion $= \frac{k_1}{s-j\omega} + \frac{k_2}{s+j\omega} + \left(F_s(s) = \frac{k_3}{\tau s+1} \right)$

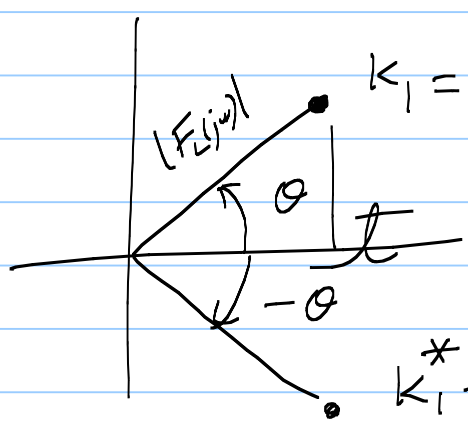
this will decay to zero as $t \to \infty$

$k_1 = (s-j\omega) Y(s) \Big|_{s=j\omega} = \frac{j\omega}{2j\omega} F_L(j\omega) = \frac{F_L(j\omega)}{2}$

$k_2 = (s+j\omega) Y(s) \Big|_{s=-j\omega} = \frac{(-j\omega)}{-2j\omega} F(-j\omega) = \frac{F_L(-j\omega)}{2}$

$= k_1^*$

$\tan\theta = \frac{Im(F_L(j\omega))}{Re[F_L(j\omega)]}$



$k_1 = \frac{F_L(j\omega)}{2} = |F(j\omega)| e^{j\theta}$

$k_1^* = k_2 = \frac{F_L(-j\omega)}{2} = |F(j\omega)| e^{-j\theta}$

Take $\mathcal{L}^{-1}$ of ①

$$y(t) = k_1 e^{j\omega t} + k_1^* e^{-j\omega t} + \mathcal{L}^{-1}\left[\frac{k_3}{\tau s+1}\right]$$

(where $\mathcal{L}^{-1}\left[\frac{k_3}{\tau s+1}\right] = \frac{k_3}{\tau} e^{-\frac{1}{\tau}t}$)

$$= \frac{|F_L(j\omega)|}{2} e^{j\theta} \cdot e^{j\omega t} + \frac{|F_L(j\omega)|}{2} e^{-j\theta} \cdot e^{-j\omega t} + \frac{k_3}{\tau} e^{-\frac{t}{\tau}}$$

$$= |F_L(j\omega)| \left[\underbrace{\frac{e^{j(\omega t+\theta)} + e^{-j(\omega t+\theta)}}{2}}_{\cos(\omega t+\theta)}\right] + \frac{k_3}{\tau} e^{-\frac{t}{\tau}}$$

$$= |F_L(j\omega)| \cos(\omega t+\theta) + \underbrace{\frac{k_3}{\tau} e^{-\frac{t}{\tau}}}_{\text{transient part}}$$

Steady state response

$$y_{ss}(t) = |F_L(j\omega)| \cos(\omega t+\theta)$$

where $\theta = \tan^{-1}\left[\frac{Im[F_L(j\omega)]}{Re[F_L(j\omega)]}\right]$

input $r(t) = \cos \omega t$ — ②

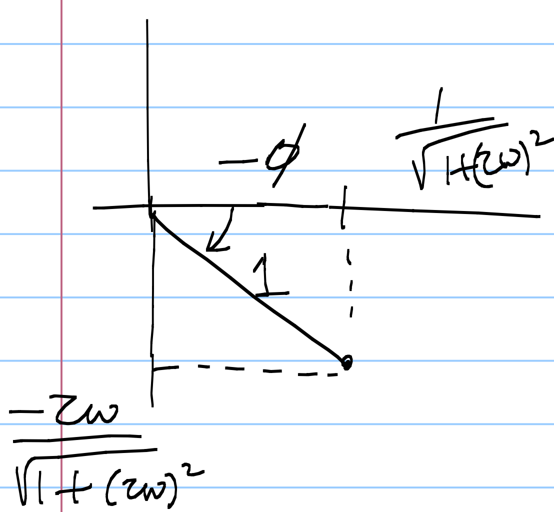
memorize this

use the result ②

$$y_{ss}(t) = \left|\frac{1}{\tau s+1}\right|_{s=j\omega} \cos(\omega t+\theta)$$

$$F(j\omega) = \frac{1}{j\tau\omega+1} = \frac{1-\tau\omega j}{1+(\tau\omega)^2} = \frac{1}{\sqrt{1+(\tau\omega)^2}}\left[\frac{1}{\sqrt{1+(\tau\omega)^2}} - \frac{\tau\omega}{\sqrt{1+(\tau\omega)^2}}\,j\right]$$

$$= \qquad = \frac{1}{\sqrt{1+(\tau\omega)^2}}\,e^{-\phi j}$$



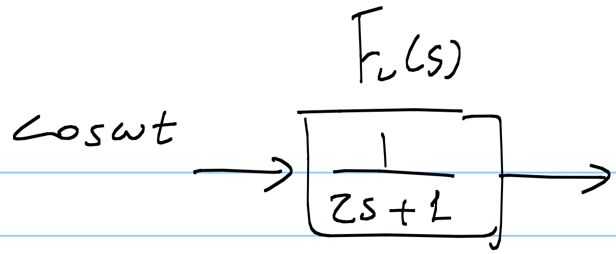
$$\phi = \tan^{-1}[\tau\omega]$$

a) so $$y_{ss}(t) = \frac{1}{\sqrt{1+(\tau\omega)^2}} \cos(\omega t-\phi)$$

magnitude $= M(\omega)\cos(\omega t-\phi(\omega))$

b) when ω is small (low Frequency)
$\rightarrow M(\omega) \approx 1$
when ω is big $\rightarrow M(\omega) \simeq 0$ (High Freq)

Thus only low frequency parts of input pass ~~the~~ filter $\longrightarrow$ lowpass filter



$$y_{ss}(t) = |F_L(j\omega)| \cos(\omega t + \phi)$$