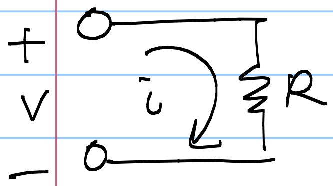
Electric Circuits

$\mathcal{L}$

Resistor

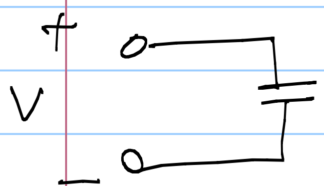


$v = iR$

$$\underset{\text{cause}}{I(s)} \longrightarrow \boxed{R} \longrightarrow \underset{\text{effect}}{V(s)}$$

$\therefore V(s) = R\,I(s)$

Capacitor



$q = CV$

$i = C\dfrac{dv}{dt}$

$$I(s) \longrightarrow \boxed{\frac{1}{Cs}} \longrightarrow V(s)$$

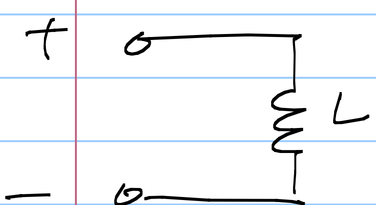
$V = \dfrac{1}{C}\displaystyle\int_0^t i\,dt + V_0$

$\mathcal{L}$

$V_0 = 0$

$V(s) = \dfrac{1}{Cs} I(s)$

Inductor



$V = L\dfrac{di}{dt}$

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$$I(s) \longrightarrow \boxed{LS} \longrightarrow V(s)$$

$V(s) = SL\,I(s)$

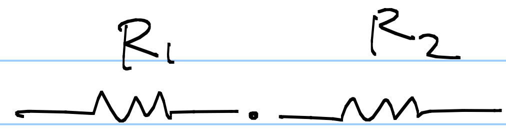
Impedence

Take $\mathcal{L}$ of elements

$V(s) = Z(s)\,I(s)$

$$I(s) \longrightarrow \boxed{Z(s)} \longrightarrow V(s)$$

	time domain	s-domain
R	$v = iR$	$V(s) = R\, I(s)$ — $Z(s)$
L	$v = L\frac{di}{dt}$	$V(s) = sL\, I(s)$ — $Z(s)$
C	$v = \frac{1}{c}\int i\,dt$	$V(s) = \frac{1}{cs} I(s)$ — $Z(s)$



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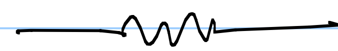
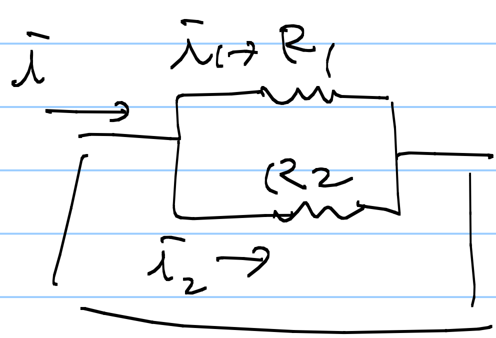
$R = R_1 + R_2$





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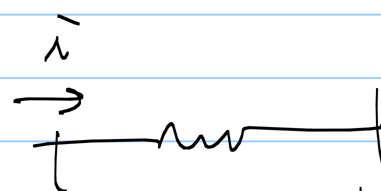


$R = \left[\frac{1}{R_1} + \frac{1}{R_2}\right]^{-1}$

V

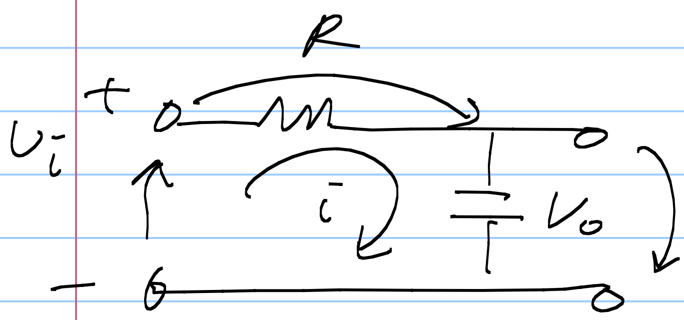
$$V = Ri = R_1 i_1 = R_2 i_2$$

$$i_1 = \frac{V}{R_1} \qquad i_2 = \frac{V}{R_2}$$

$$\cancel{V} = Ri = R(i_1 + i_2)$$

$$= R\cancel{V}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Low-pass filter



KCs Voltage Law

$$V_i - iR - V_o = 0$$

$$V_o = \frac{1}{C}\int_0^t i\,dt$$

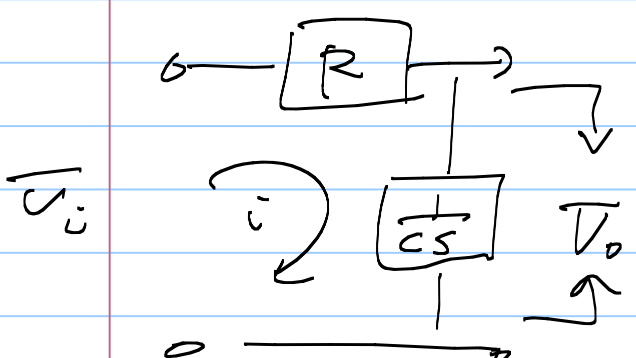
$$i = \frac{C\,dV_o}{dt}$$

$$V_i - \left(RC \cdot \frac{dV_o}{dt}\right) - V_o = 0$$

$\mathcal{L}$

$$V_i(s) - RC\,s\,V_o - V_o(s) = 0$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{(RCs+1)}$$



$$V_o(s) = \boxed{\frac{1}{CS}}\, I(s)$$

$$V_i(s) = \left(R + \frac{1}{CS}\right) I(s)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{CS}}{R + \frac{1}{CS}} = \frac{1}{RCS+1}$$

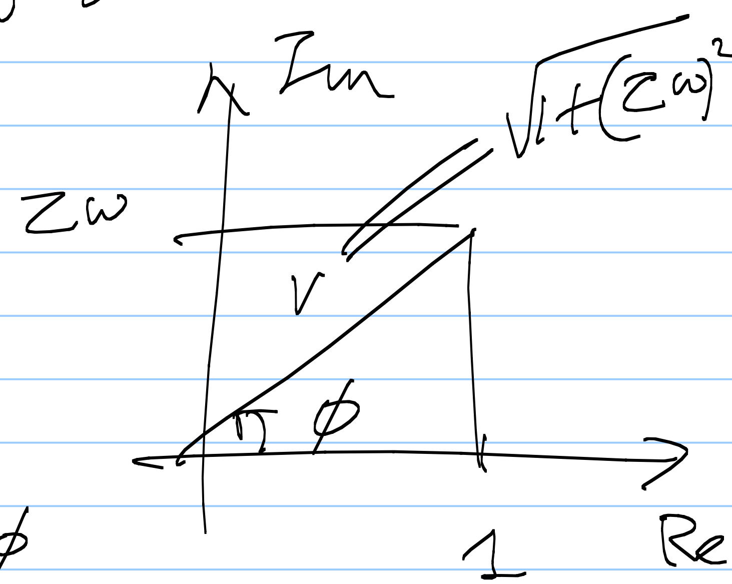
$$V_i \longrightarrow \boxed{H(s)} \longrightarrow v_o$$

Take $v_i = e^{j\omega t} = \cos\omega t + j\sin\omega t$

$$v_o = H(j\omega)\, e^{j\omega t}$$

$$= \frac{1}{RS(j\omega)+1}\, e^{j\omega t}$$

$\tau = RS$



$$\Rightarrow \quad \tau j\omega + 1$$

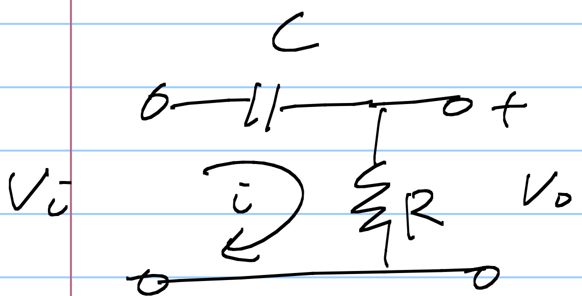
$$\parallel$$

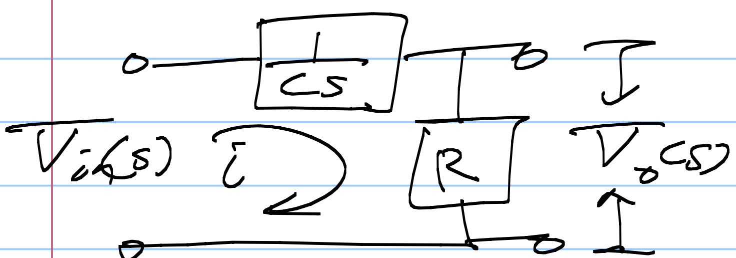
$$\Rightarrow \quad \sqrt{1+(\tau\omega)^2}\, e^{j\phi}$$

$$= \frac{e^{-j\phi}}{\sqrt{1+(\tau\omega)^2}}\, e^{j\omega t}$$

$$= \frac{1}{\sqrt{1+\tau^2\omega^2}} \left(e^{j(\omega t-\phi)} = \cos(\omega t-\phi) + j\sin(\omega t-\phi) \right)$$

High-pass-filter





$$V_o = R\, I(s)$$

$$V_i = \left(\frac{1}{Cs} + R\right) I(s)$$

k's Voltage law

$$\frac{V_o}{V_i} = \frac{R}{\frac{1}{Cs} + R} = \frac{RCs}{RCs+1}$$

$H(s)$

$$v_i = e^{j\omega t} = \cos\omega t + j\sin\omega t$$

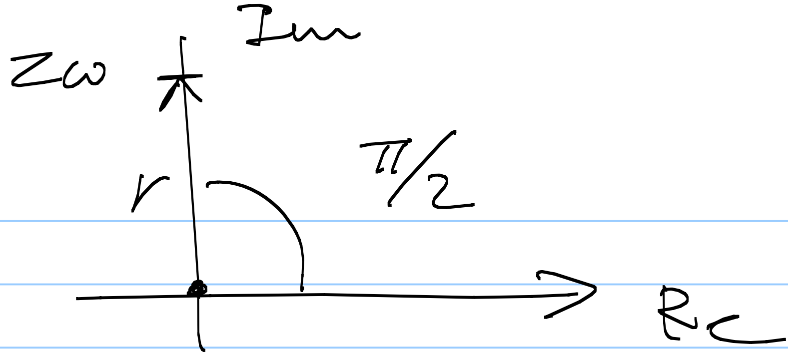
$$v_o = H(j\omega)\, e^{j\omega t}$$

$$= \frac{RCj\omega}{RCj\omega + 1} e^{j\omega t} = \frac{\tau\omega\, e^{j\pi/2}\, e^{j\omega t}}{\sqrt{1+\tau^2\omega^2}\, e^{j\phi}}$$

τ

$$RCj\omega + 1 = \sqrt{1+\tau^2\omega^2}\, e^{j\phi}$$

$$\tau = RC$$



$$= z\omega e^{\pi/2\, j}$$

$$v_0 = \frac{z\omega}{\sqrt{1+z^2\omega^2}} e^{j(\omega t + \pi/2 - \phi)}$$

$$= \frac{z\omega e^{j\pi/2}}{\sqrt{1+z^2\omega^2}\, e^{j\phi}} e^{j\omega t}$$

$$= \frac{z\omega}{\sqrt{1+z^2\omega^2}} \left[\cos(\omega t + \pi/2 - \phi) + j \sin(\omega t + \pi/2 - \phi) \right]$$