Transfer Function

- Defined as the ratio of the Laplace transform of the output signal to that of the input signal (think of it as a gain factor!)
- Contains information about dynamics of a Linear Time Invariant system
- Time domain
  \[ u(t) \rightarrow h(t) \rightarrow y(t) = h(t) \ast u(t) \]
- Frequency domain
  \[ U(s) \rightarrow H(s) \rightarrow Y(s) = H(s)U(s) \]

Mass-Spring-Damper System

- ODE
  \[ M\ddot{y}(t) + b\dot{y}(t) + ky(t) = u(t) \]
- Assume all initial conditions are zero. Then take Laplace transform,
  \[
  \begin{align*}
  Ms^2Y(s) + bsY(s) + kY(s) &= U(s) \\
  \frac{Y(s)}{U(s)} &= \frac{1}{Ms^2 + bs + k}
  \end{align*}
  \]
  Transfer function
**Transfer Function**

- Differential equation replaced by algebraic relation \( Y(s) = H(s)U(s) \)
- If \( U(s) = 1 \) then \( Y(s) = H(s) \) is the impulse response of the system
- If \( U(s) = 1/s \), the unit step input function, then \( Y(s) = H(s)/s \) is the step response
- The magnitude and phase shift of the response to a sinusoid at frequency \( \omega \) is given by the magnitude and phase of the complex number \( H(j\omega) \)

\[
\mathcal{L}[\delta(t)] = \int_{0}^{\infty} \delta(t)e^{-st}dt = 1
\]

\[
\mathcal{L}[1(t)] = \int_{0}^{\infty} e^{-st}dt = \frac{1}{s}
\]

**Kirchhoff’s Voltage Law**

- The algebraic sum of voltages around any closed loop in an electrical circuit is zero.

\[ v_1 + v_2 + v_3 = 0 \]
**Kirchhoff's Current Law**

- The algebraic sum of currents into any junction in an electrical circuit is zero.

\[ i_1 + i_2 + i_3 = 0 \]