2nd order system $(0 < \zeta < 1)$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2} = \frac{1}{\left(\frac{s}{\omega_n}\right)^2 + 2\zeta\left(\frac{s}{\omega_n}\right) + 1}$$

$$|G(j\omega)|_{dB} = 20\log\left|\frac{1}{\left(j\frac{\omega}{\omega_n}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + 1}\right|$$

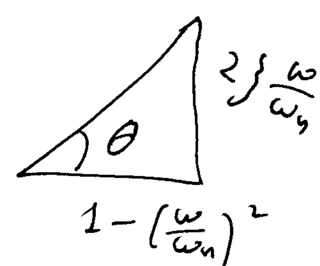
$$= -20\log\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right] + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}$$

i) $\frac{\omega}{\omega_n} \ll 1$: $M_{dB} \to 0$

ii) $\frac{\omega}{\omega_n} \to \infty$: $M_{dB} \simeq -20\log_{10}\underbrace{\left(\frac{\omega}{\omega_n}\right)^2}_{\text{dominant term.}}$

phase

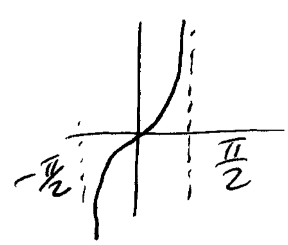
$$\phi = \angle\left[\frac{1}{\left(j\frac{\omega}{\omega_n}\right)^2 + \left(2\zeta\frac{\omega j}{\omega_n}\right) + 1}\right] = -\tan^{-1}\left[\frac{\left(2\zeta\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

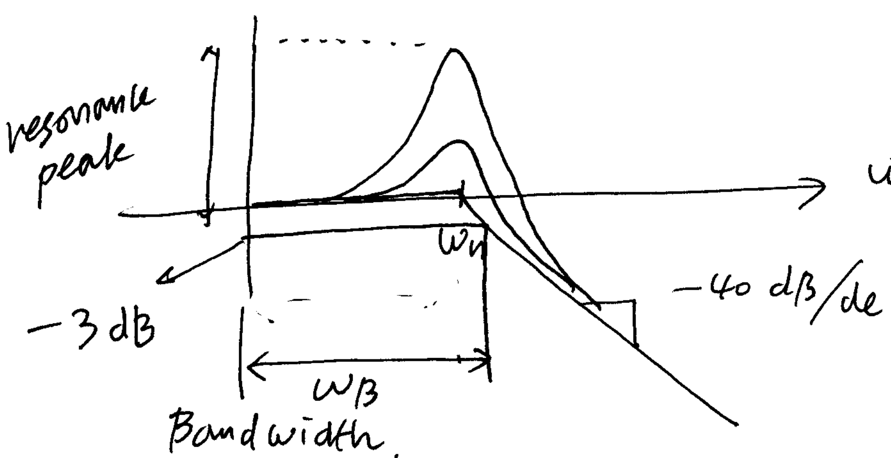


i) $\frac{\omega}{\omega_n} \approx 0$ $\quad \phi = -\tan^{-1}(0) = 0$

ii) $\frac{\omega}{\omega_n} = 1$ $\quad \phi = -\tan^{-1}\left(\frac{2\zeta}{0}\right) = -90°$

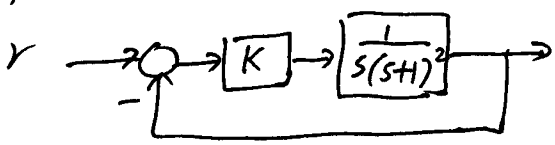
iii) $\frac{\omega}{\omega_n} \to \infty$ $\quad \phi = -\tan^{-1}(0^-) = -180°$

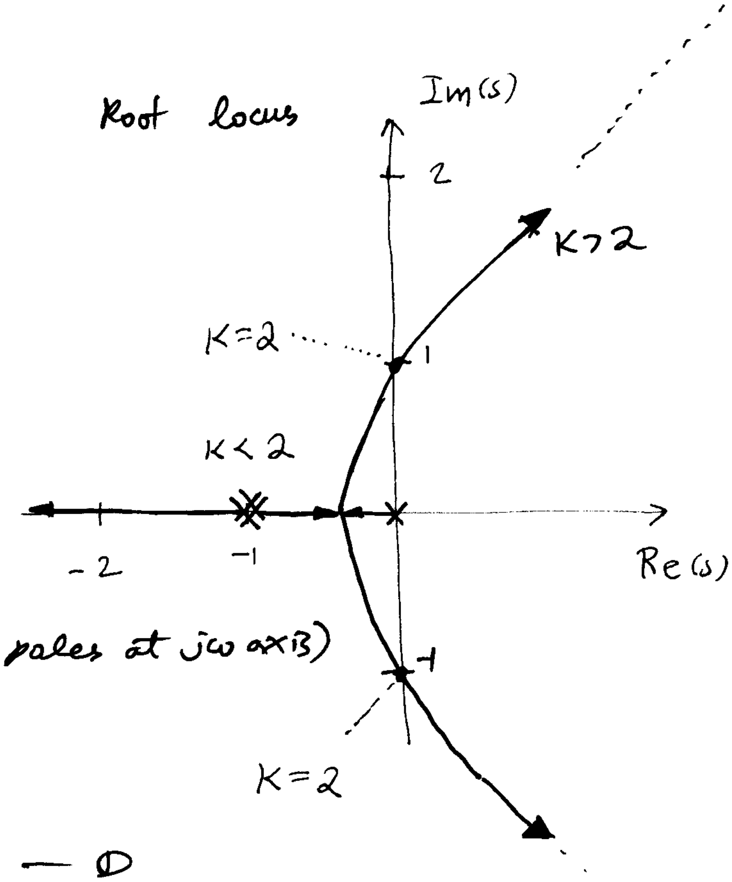




Stability

ex)





$$G(s) = \frac{1}{s(s+1)^2}$$

C.L. system is marginally stable at $K = 2$ with $\pm j$ (two closed loop poles at $j\omega$ axis)

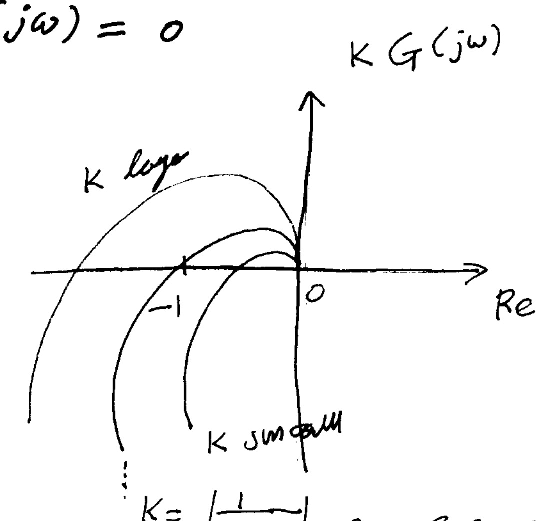
Consider C.E.

$$1 + k\,G(s) = 0 \quad \text{— ①}$$

when marginally stable case $j\omega$ is the closed loop poles

C.E. ① $\Rightarrow$ $1 + K\,G(j\omega) = 0$

$$\begin{cases} \angle KG(j\omega) = -180^\circ \\ |K\,G(j\omega)| = 1 \end{cases}$$



$K_g = \left|\frac{1}{G(j\omega_0)}\right|$ & $\angle G(j\omega_0) = 180^\circ$

Consider a minimum phase system (all poles & zeros $\in$ LHP of $G(s)$)

$\boxed{\text{phase margin } \gamma}$ is the additional phase lage at the gain crossover frequency ω_g required to bring the system to instability.

Where ω_g: defined by $|G(j\omega_g)| = 1$

so
$$\gamma = 180° + \underbrace{G(j\omega_g)}_{\text{negative angle}}$$

~~[illegible]~~ stable $G(s)$ all poles & zeros $\in$ LHP

$$\Rightarrow \quad \gamma > 0$$

Gain margin
$$K_g = \left| \frac{1}{G(j\omega_p)} \right|$$

where
$$G(j\omega_p) = -180°$$

$\omega_p \rightarrow$ phase cross over freq.

$$K_g \, dB = 20 \log K_g = -20 \log |G(j\omega_p)|$$

For a stable minimum phase system, K_g indicates how much the gain can be increased before the system becomes unstable.

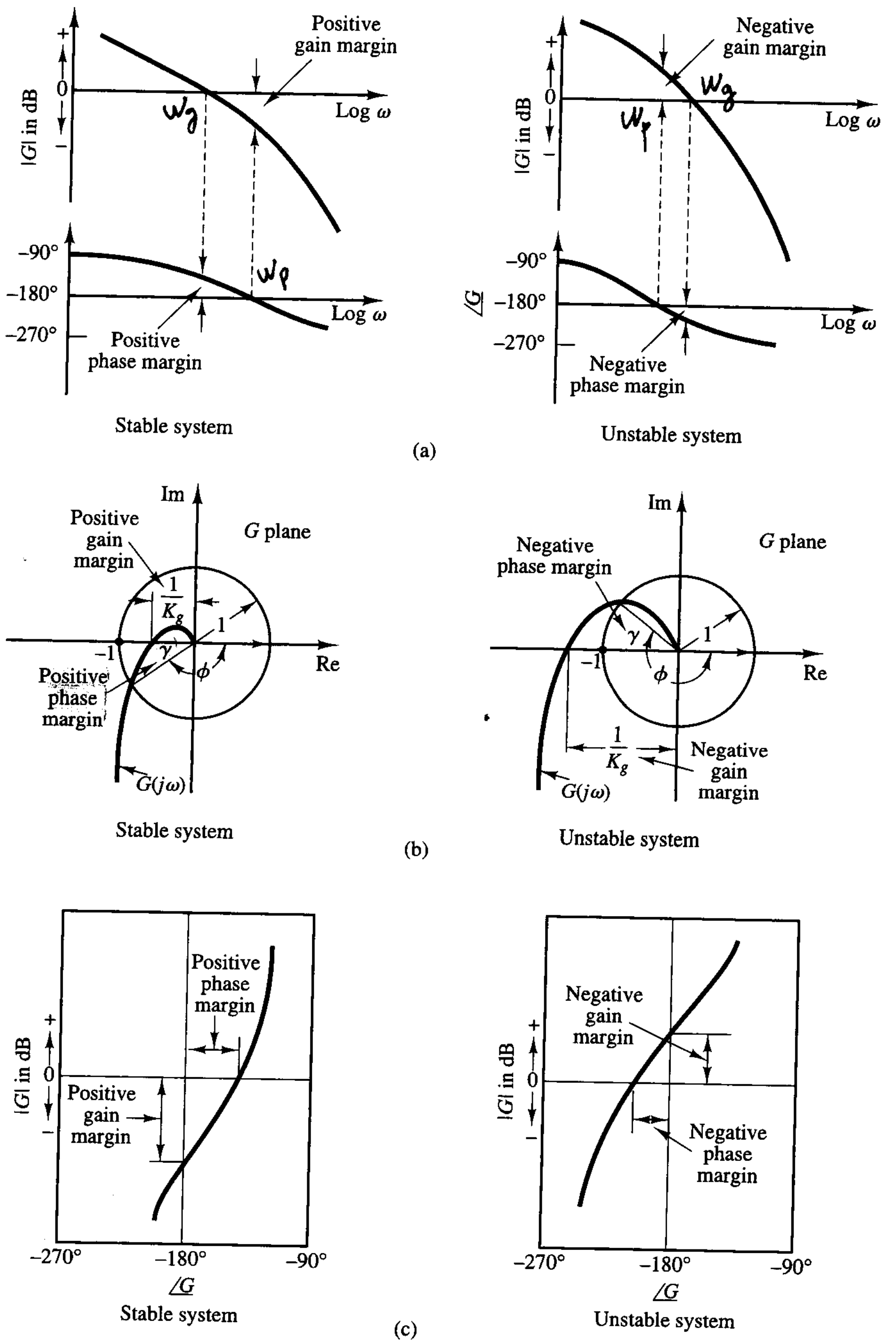


Figure 8–75
Phase and gain margins of stable and unstable systems.
(a) Bode diagrams;
(b) polar plots;
(c) log-magnitude versus phase plots.