Partial-fraction Expansion (Text, page 637-641)

- F(s) is rational, $m \leq n \longrightarrow$ realizable condition (d/dt is not realizable)

$$F(s) = \frac{b_m s^m + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$$

$$= \frac{b_m \prod_{i=1}^{m}(s - z_i)}{\prod_{i=1}^{n}(s - p_i)}$$

zeros: z_i

poles: p_i

$$= \frac{k_1}{s - p_1} + \frac{k_2}{s - p_2} + \cdots + \frac{k_n}{s - p_n}$$

$$f(t) = \sum_{i=1}^{n} k_i e^{p_i t} 1(t)$$

Cover-up Method

$$F(s) = \frac{k_1}{s - p_1} + \frac{k_2}{s - p_2} + \cdots + \frac{k_n}{s - p_n}$$

$$(s - p_i)F(s) = \frac{k_1(s - p_i)}{s - p_1} + k_i + \cdots + \frac{k_n(s - p_i)}{s - p_n}$$

$$\boxed{k_i = (s - p_i)F(s)|_{s=p_i}}$$

- Check the repeated root for the partial-fraction expansion (page 638)

Example

- Obtain y(t)? $\ddot{y}(t) - y(t) = t, \quad y(0) = 1, \dot{y}(t) = 1$

$$s^2 Y(s) - sy(0) - \dot{y}(0) - Y(s) = \frac{1}{s^2},$$

$$\begin{aligned} Y(s) &= \frac{1}{s-1} + \frac{1}{s^2(s^2-1)} \\ &= \frac{1}{s-1} + \frac{s^2-(s^2-1)}{s^2(s^2-1)} = \frac{1}{s-1} + \boxed{\frac{1}{s^2-1}} - \frac{1}{s^2} \\ &= \frac{1}{s-1} + \boxed{\frac{1}{2}\frac{1}{s-1} - \frac{1}{2}\frac{1}{s+1}} - \frac{1}{s^2} \end{aligned}$$

$$y(t) = \mathcal{L}^{-1}(Y(s)) = \left[\frac{3}{2}e^{t} - \frac{1}{2}e^{-t} - t\right] 1(t)$$