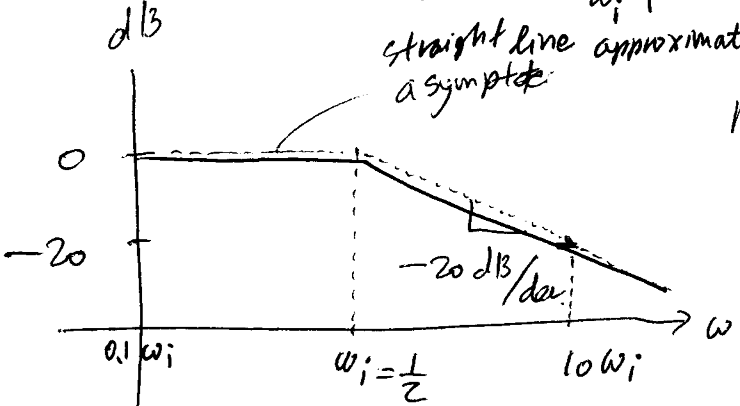
(3) Nonzero Real Poles & Zeros.

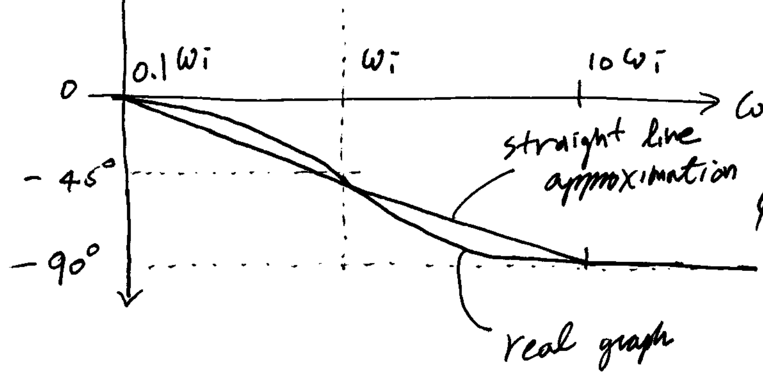
$$G(s) = \frac{1}{\left(1+\frac{s}{\omega_i}\right)} = \frac{1}{(\tau s+1)} \quad \omega_i = \frac{1}{\tau}$$

Low pass filter

$$M_{dB} = 20\log|G(j\omega)| = 20\log\left|\frac{1}{1+\frac{j\omega}{\omega_i}}\right| = -20\log\sqrt{\left(\frac{\omega}{\omega_i}\right)^2+1}$$



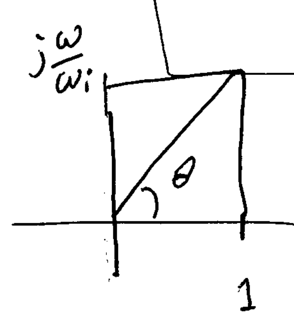
M_{dB}
① $\omega \le \omega_i$
$M_{dB} \simeq -20\log 1$
② $\omega > \omega_i$
$M_{dB} \approx -20\log\frac{\omega}{\omega_i}$
$-20\log\omega + 20\log\omega_i$



$\phi =$
① $\omega \le \omega_i$
$\phi \simeq -\angle 1 = 0$
② $\omega > \omega_i$
$\phi \simeq -\angle \frac{j\omega}{\omega_i}$
$= -90°$

for asymptote

$$\phi = \angle G(j\omega) = \angle\frac{1}{1+\frac{j\omega}{\omega_i}}$$
$$= -\angle\left(1+\frac{j\omega}{\omega_i}\right)$$
$$= -\angle \tan^{-1}\left(\frac{\omega}{\omega_i}\right)$$

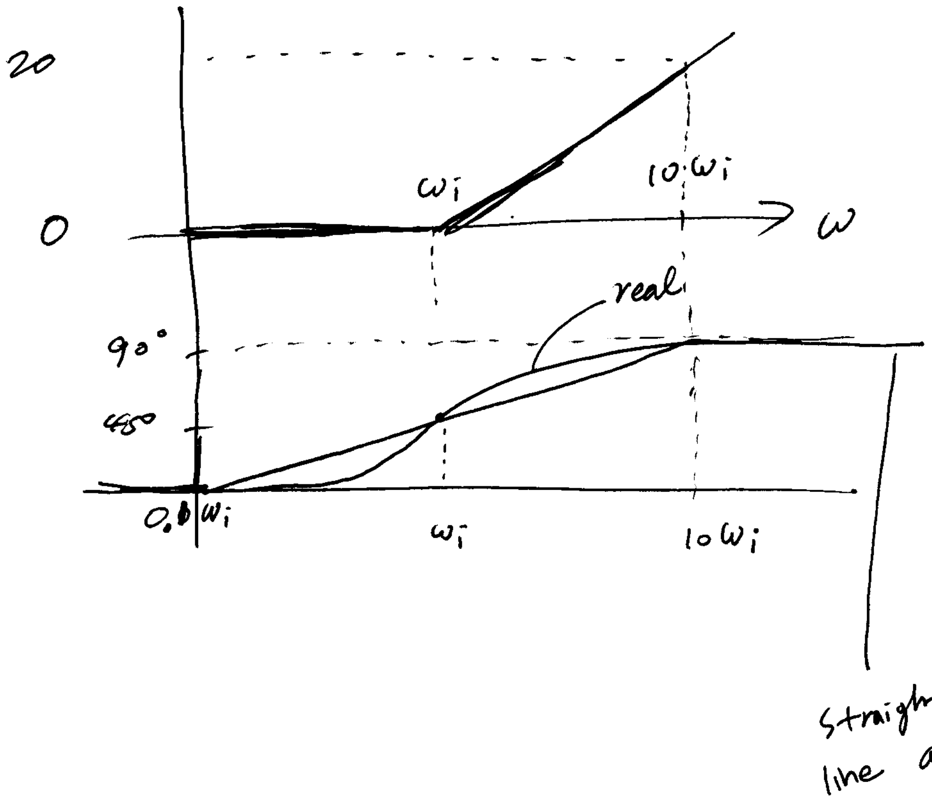


$$\tan\theta = \frac{\omega}{\omega_i}$$

$G(s) = (1 + \frac{s}{\omega_i})$ High pass filter

$$M_{dB} = 20\log\left|1 + \frac{j\omega}{\omega_i}\right| = 20\log\sqrt{\left(\frac{\omega}{\omega_i}\right)^2 + 1}$$

$$\phi = \angle G(j\omega) = \angle\left(1 + \frac{j\omega}{\omega_i}\right) = \tan^{-1}\left(\frac{\omega}{\omega_i}\right)$$



$\omega \leq \omega_i$

$M_{dB} \approx 20\log 1 = 0$

$\phi \approx \angle 1 = 0$

$\omega > \omega_i$

$M_{dB} \approx 20\log\left(\frac{\omega}{\omega_i}\right)$

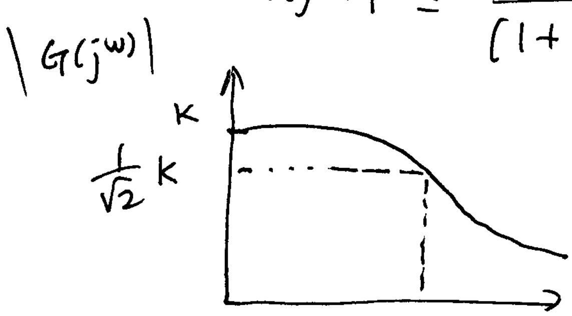
$= \underline{20\log\omega} - 20\log\omega_i$

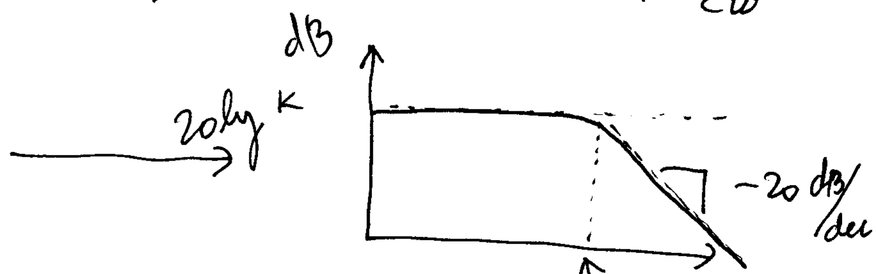
$\phi \approx \angle\frac{j\omega}{\omega_i} = 90°$

Bandwith of $G(s)$ $\qquad G(s) = \frac{K}{\tau s + 1}$

$$G(j\omega) = \frac{K}{1 + j\tau\omega} = |G(j\omega)|\, e^{j\phi(\omega)}$$

$$|G(j\omega)| = \frac{K}{[1 + \tau^2\omega^2]^{1/2}}, \qquad \phi(\omega) = -\tan^{-1}\tau\omega$$





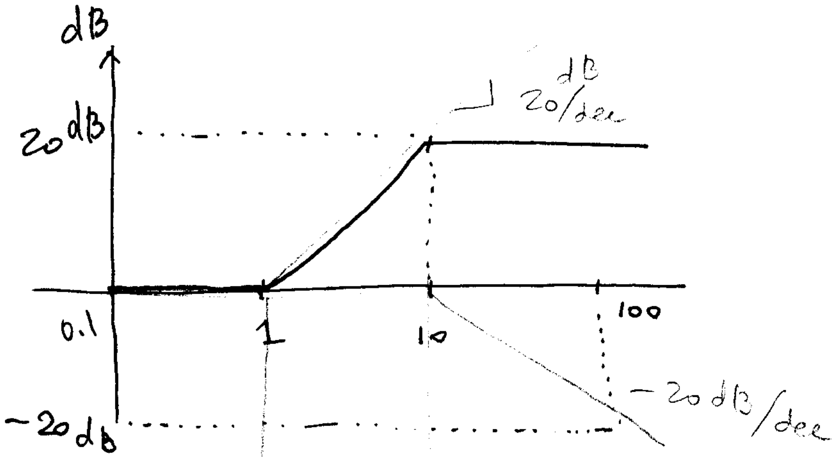
System Bandwidth

$\rightarrow \omega_B$: frequency at which the gain is equal to $1/\sqrt{2}$ of DC gain $\left(\frac{K}{\sqrt{1+\tau^2\omega_B^2}} = \frac{K}{\sqrt{2}}\right)$

$\omega_B = 1/\tau$

ex) $G(s) = \frac{10(s+1)}{s+10} = \frac{s+1}{(s/10+1)}$

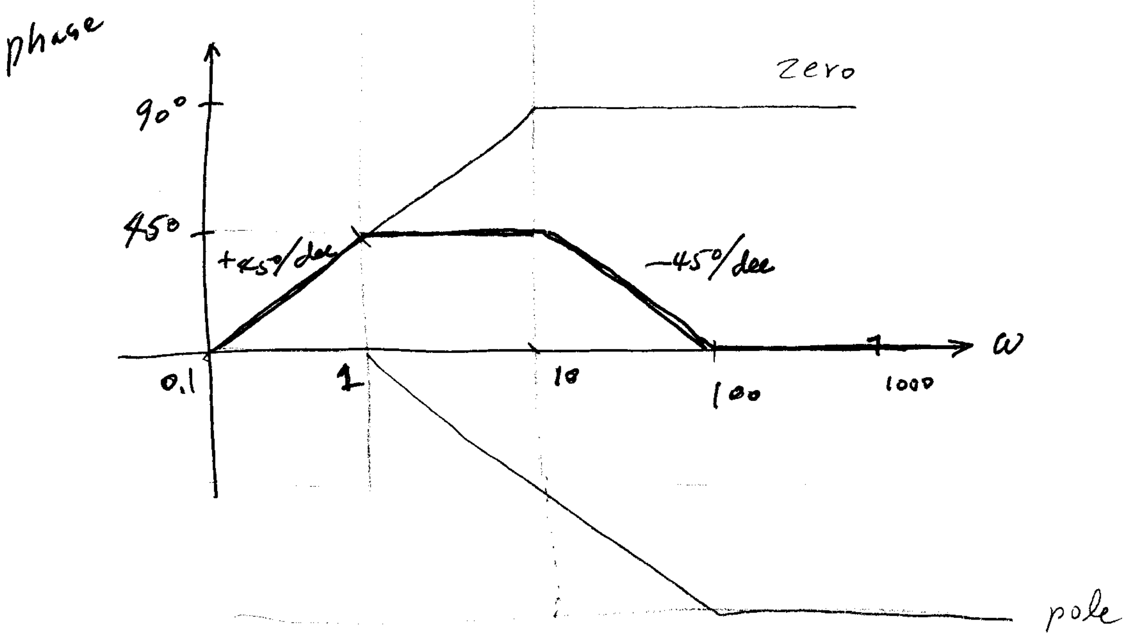
$G(j\omega) = \frac{j\omega+1}{\frac{j\omega}{10}+1}$



① — $\lim_{\omega\to0} G(j\omega) = 1$

② — $\lim_{\omega\to\infty} G(j\omega) = 10 \Rightarrow 20dB$

$$20\log\left(\left|\frac{j\omega+1}{j\frac{\omega}{10}+1}\right|\right) = 20\log(|j\omega+1|) - 20\log\left|j\frac{\omega}{10}+1\right|$$



from ① → 0°

$\lim_{\omega\to0} \angle G(j\omega) = 0°$

from ②

$\lim_{\omega\to0} \angle G(j\omega) = 0°$