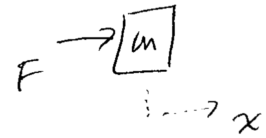
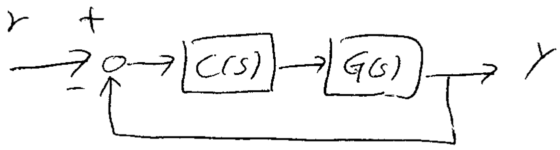
phase Lead

ex) $G(s) = \frac{1}{s^2}$ satellite



$m\ddot{x} = F$

double integrator



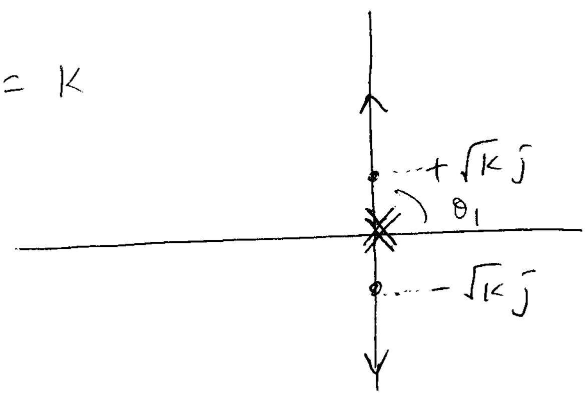
① if $C(s) = K$

$1 + \frac{K}{s^2} = 0 \rightarrow -2\theta_1 = \pm 180°$

$\theta_1 = \pm 90°$

$s_{1,2} = \pm\sqrt{K}j$

marginally stable $K > 0$



② $C(s) = \frac{K_c(s-z_0)}{(s-p_0)}$

$1 + \underbrace{\frac{K_c(s-z_0)}{(s-p_0)}}\frac{1}{s^2} = 0$

① phase condition

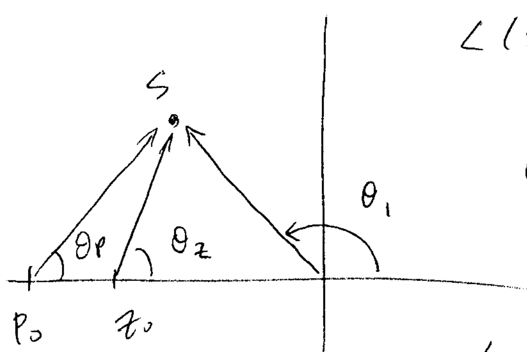
$\angle \frac{(s-z_0)}{(s-p_0)}\frac{1}{s^2} = -\angle\frac{1}{K_c}$

$\angle(s-z_0) - \angle(s-p_0) - 2\angle s = \pm 180°$

$\underbrace{\theta_z - \theta_p} - 2\theta_1 = -180°$

positive phase gain $= \phi > 0$

(phase lead)



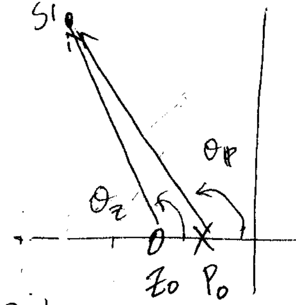
$\theta_1 = \underbrace{90° + \frac{\phi}{2}}$ ↑

push the C.L. poles to the left $\Rightarrow$ stabilizing effect

better transient effect.

phase Lag design

$$K\,C(s) = K \underbrace{\left[\frac{K_c\,(s - z_0)}{(s - P_0)}\right]}$$



$|z_0| > |P_0|$

Assume DC gain = 1 $\Rightarrow \dfrac{K_c z_0}{P_0} = 1$

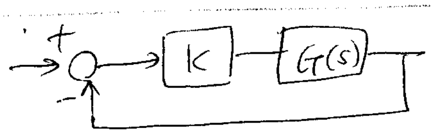
pro ① To improve steady-state response

$(\theta_z - \theta_p = \phi < 0)$

Cons ② $C(s)$ adds a negative angle, tends to shift the root rolcus to the right in the s-plane.

$\Rightarrow$ make $|z_0| \approx |P_0| \Rightarrow \theta_z \approx \theta_p$ $\phi \approx 0, < 0$

minimize the destabilizing effect!



$\Rightarrow 1 + K\,G(s) = 0 \Rightarrow K_0 = \dfrac{-1}{G(s_1)}$

Step 1. Choose K_0 to yield the value of the desired closed-loop pole s_1 in uncompensated system.

step 2. Now use $K\,C(s) = K \left[\dfrac{K_c\,(s - z_0)}{(s - P_0)}\right]$

The gain K required to place a root of the locus at $\approx s_1$ for the compensated system is

C.E. $1 + K C(s_1)\,G(s_1) = 0$

$$K = \frac{-1}{C(s_1)\,G(s_1)} = +\frac{1}{C(s_1)} \cdot K_0 \approx \frac{K_0}{K_c}$$

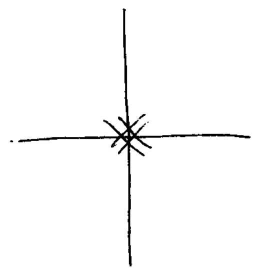
why ?

$$\frac{K_c\,(s_1 - z_0)}{(s_1 - P_0)} \approx K_c$$

Assume $\boxed{C(0) = \dfrac{K_c z_0}{P_0} = 1}$

Calculate K for the desired steady-state response.

Step 3. Choose $|z_0| < |s_1|$ Step 4. obtain $P_0 = K_c z_0$



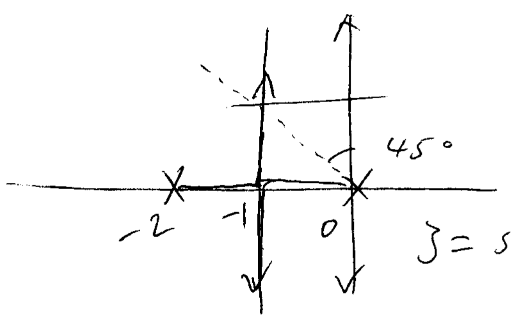
phase-lag compensation to the satellite control ?

(No)

We have to stabilize it first.

(use phase lead first)

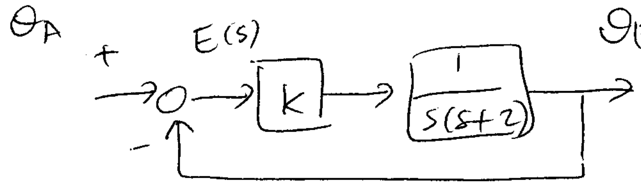
ex) 7.15 $\quad G(s) = \frac{1}{s(s+2)} \qquad 1 + K\,G(s) = 0$



$P_{RK} \quad s_1 = -1 + j$

$\therefore \quad K_0 = 2$

$\zeta = \sin 45°$



unit ramp

For $\frac{1}{s^2} = \theta_A$

steady state error = 0.2

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s\,E(s) = \lim_{s \to 0} s \cdot \frac{1}{s^2} \frac{1}{1 + \frac{K}{s(s+2)}} =$$

$$= \frac{1}{s + \frac{K}{s+2}} = \frac{2}{K} = \frac{2}{10}$$

$K = 10$

Step 2. $\quad K_c = \frac{P_0}{Z_0} = \frac{K_0}{K} = \frac{2}{10} = 0.2$

Step 3 $\quad$ choose $\quad Z_0 = -0.1 \ll |s_1| = \sqrt{2}$

Step 4 $\quad \rightarrow \quad P_0 = K_c Z_0 = -0.02$

$$K\,C(s) = K \cdot \frac{K_c (s - Z_0)}{(s - P_0)} = 10 \cdot 0.2 \frac{(s+0.1)}{s+0.02} = \frac{2(s+0.1)}{(s+0.02)}$$