Controller Design

I. Effects of addition of zeros (P-D controller) \( k = (k_p + k_d s) \)

pulling the root locus to the left, tending to make
the system move stable and to speed up the settling of
the response.

→ **Stabilizing effect** → **Better transient response**

\[ C(s) = \frac{1}{(s-p_1)(s-p_2)(s-p_3)} \]

\[ C(s) = k_d (s-z_1) \]

**PD controller with \( z_1 < 0 \)**

\[ = k_p + k_d s \]
II. Effect of the addition of poles

Pulling the root locus to the right.

**Destabilizing effect** \(\rightarrow\) Slow down the settling of the response.

(Steady state error sense)

By adding \(\frac{1}{s}\) \(\rightarrow\) Perfect tracking for \(R(s) = \frac{1}{s}\)

Perfect disturbance rejection for \(D(s) = \frac{1}{s}\)

& C.L. Stable,

Perfect tracking for \(R(s) = \frac{1}{s^2}\)

Perfect disturbance rejection for \(D(s) = \frac{1}{s^2}\)

\[\text{Disturbance} \xrightarrow{D(s)} \text{Sensor noise} \]

\[\text{Sensormismatch offset} = \frac{1}{s}\]

\[\text{Command} R(s) \rightarrow \text{Input} \rightarrow \text{Controller C(s)} \rightarrow \text{Gain G} \rightarrow \text{Output}\]

E.g.) PI controller,

\[C(s) = \frac{K_p + K_i}{s} = \frac{(sK_p + K_i)}{s} \rightarrow \text{zero pole at } 0\]

\[\text{Correct steady state error} \quad \frac{\text{stabilizing effect}}{\text{PD}} \quad \Rightarrow \quad \text{PID}\]

\[C(s) = K_p + \frac{K_s}{s} + K_d s\]
**Lead Compensator**

\[ C(s) = \frac{K_c (s-z_0)}{(s-p_0)} \]

**Approximate PD Controller**

Stabilizing effect.

**Controller parameterization**

\[ G(s) = \frac{1}{s(s+1)} \]

**C.E.** \[ 1 + \frac{K_c (s-z_0)}{(s-p_0)s(s+1)} = 0 \]

1. Design location of desired dominant C.L. poles:
   \[ s = -2 \pm j \left( \frac{1}{2} \right) \]

2. Select location of zero (why? 3 unknowns, 2 eqs. from C.E.):
   \[ z_0 = -2 \]

3. Utilize the angle condition:
   \[ \pm 180^\circ = \sum_{i=0}^{0} \chi (s-z_i) - \sum_{i=0}^{2} \chi (s-p_i) \]
   \[ = \chi (s+2) - \chi (s+1) - \chi (s-p_0) - \chi (s-p_2) - \chi (s) \]
   \[ = \theta_0 - \theta_1 - \theta_2 - \theta_0 \]

\[ -180^\circ = 90^\circ - \theta_0 - \left( 180^\circ - \tan^{-1}\left( \frac{2}{1} \right) \right) - (180^\circ - 45^\circ) \]

\[ 117^\circ - 90^\circ - 45^\circ = -\theta_0 \]

\[ \theta_0 = 18^\circ \]

\[ \frac{2}{l} = \tan 18^\circ \Rightarrow l = \frac{2}{\tan 18^\circ} \]

Let's pick \( p_0 = -8 \)

\[ l = -2 - p_0 = -2 - 6.155 = -8.155 \]

\[ 4 = \frac{2}{l} \]
4. Utilize the magnitude condition:

\[
\frac{\prod_{i=0}^{\Pi} |s - z_i|}{\prod_{i=0}^{\Pi} |s - p_i|} = \frac{1}{K_c}
\]

\[
k_c = \frac{M_p}{M_z} = \frac{\sqrt{b^2 + c^2} \sqrt{1 + 2^2 \sqrt{2} + 2^2}}{2} = 20
\]

\[C(s) = 20 \cdot \frac{(s+2)}{(s+8)}\]

Root locus of \(1 + K_c \frac{s+2}{(s+8)(s+4)}\) is:

Dominant poles at \(s = -1.98 \pm 2.02j\)

\(s = 0.701\)

Root locus (+[1 2], [1 9 0]) &

use matlab

![Root Locus Diagram](image)