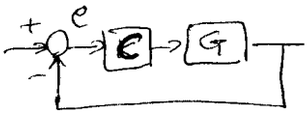
Controller Design

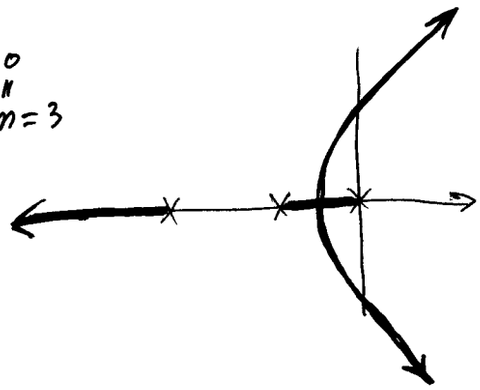


I. Effects of addition of zeros (P-D controller) $K = (K_p + K_d s)$

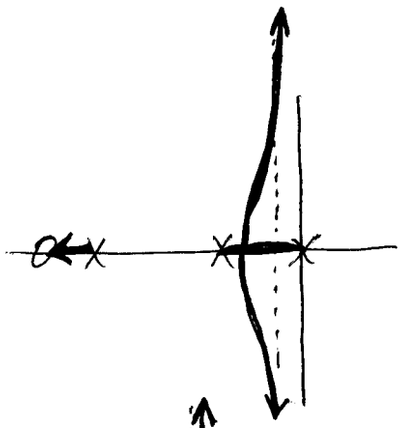
pulling the root locus to the left, tending to make the system more stable and to speed up the settling of the response.

→ $\boxed{\text{Stabilizing effect}}$ → $\boxed{\text{Better transient response}}$

$3 \;\; 0$
$\;\;\| \;\; \|$
$n - m = 3$



$3 \;\; 1$
$\;\;\| \;\; \|$
$n - m = 2$

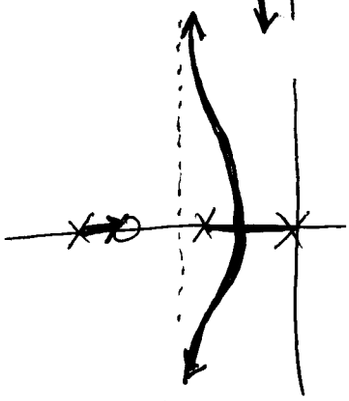


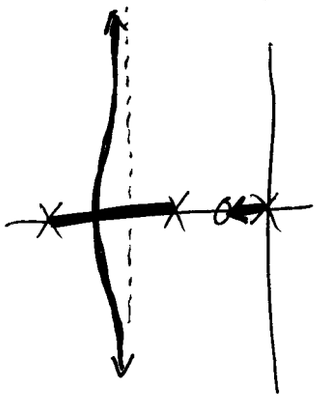
$$G(s) = \frac{1}{(s-p_1)(s-p_2)(s-p_3)}$$

$$\underbrace{C(s) = K_d(s - z_1)}$$

PD controller with $z_1 < 0$

$$= K_p + K_d s$$





II. Effect of the addition of poles

pulling the root locus to the right.

$\boxed{\text{destabilizing effect}}$ → slow down the settling of the response.

(stead state error sense)

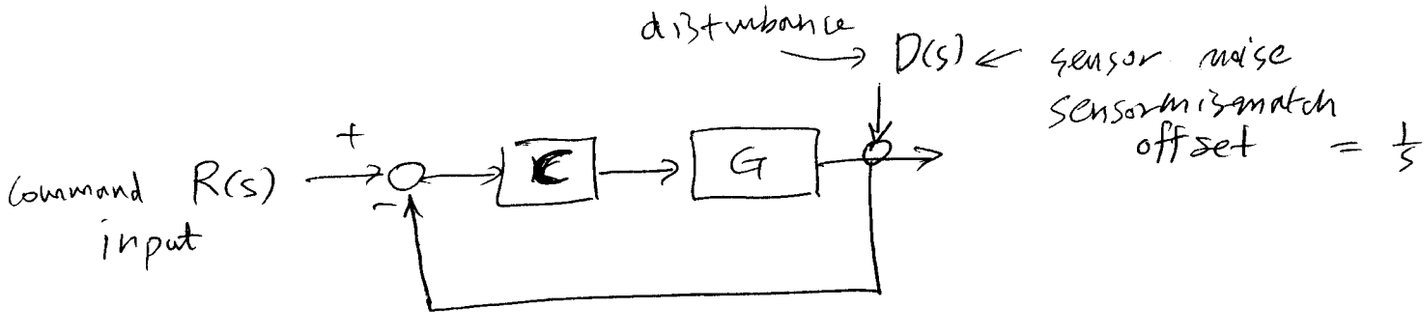
By addig $\frac{1}{s}$ → perfect tracking for $R(s) = \frac{1}{s}$

perfect disturbance input rejection for $D(s) = \frac{1}{s}$

By addig $\frac{1}{s^2}$ → Perfect tracking for $R(s) = \frac{1}{s^2}$

& C.L. stable,

perfect disturbance rejection for $D(s) = \frac{1}{s^2}$

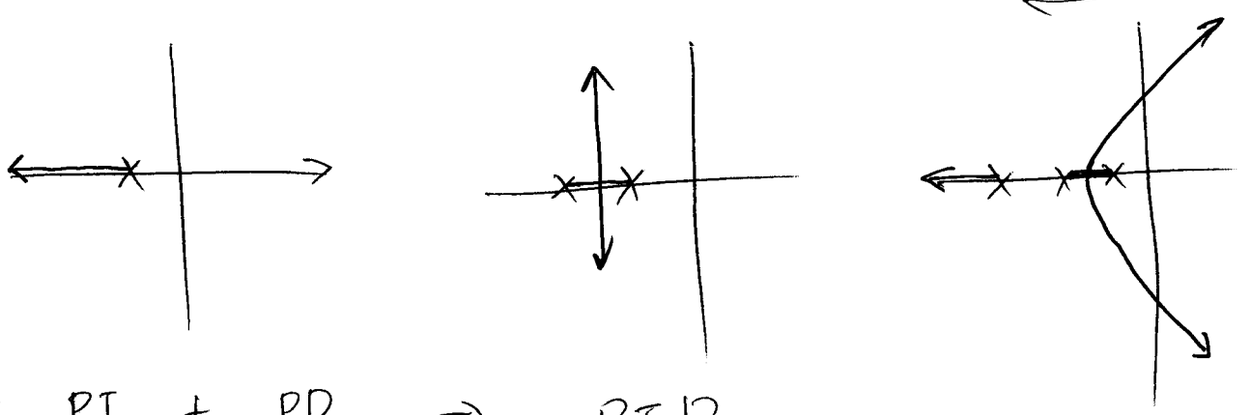


eg.) PI controller, $C(s) = K_p + \frac{K_I}{s}$

$$= \left(\frac{sK_p + K_I}{s}\right)$$

zero

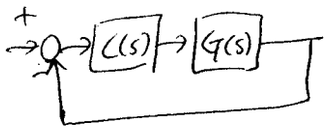
pole at 0



Pick $\underbrace{PI}_{\text{correct steady state error}}$ + $\underbrace{PD}_{\text{stabilizing effect}}$ ⇒ PID

$$C(s) = K_p + \frac{K_I}{s} + K_D s$$

Lead Compensator [Approximate PD Controller]



why? PD is NOT Causal "s"! Stabilizing effect.

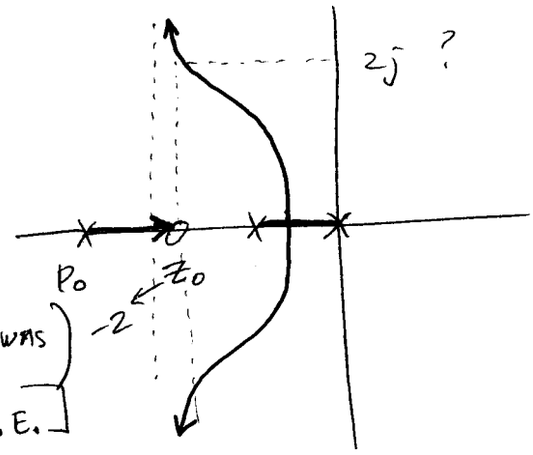
$$C(s) = K_c \frac{(s-z_0)}{(s-p_0)} \leftarrow \text{Controller parameterization}$$

$p_0,\ z_0 < 0$

$|p_0| > |z_0|$

e.g. $G(s) = \dfrac{1}{s(s+1)}$

C.E. $1 + K_c \dfrac{(s-z_0)}{(s-p_0)} \dfrac{1}{s(s+1)} = 0$



① Design location of desired dominant C.L. poles

$s = -2 \pm 2j \quad (\zeta = \frac{1}{\sqrt{2}})$

② Select location of zero (why? 3 unknowns [2 eqs. from C.E.])

$z_0 = -2$

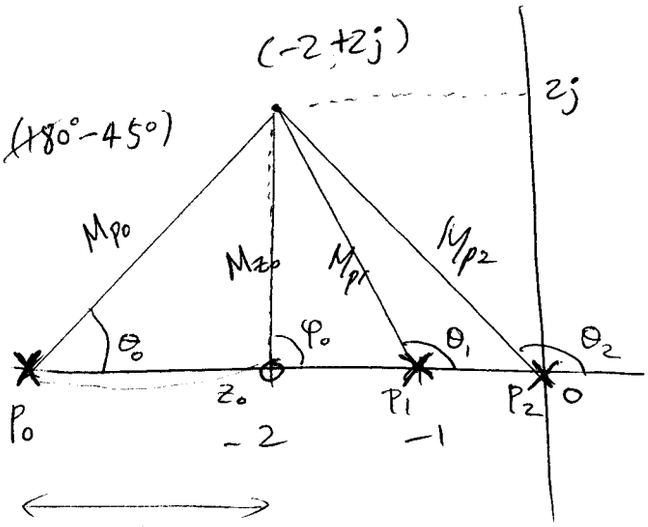
③ Utilize the angle condition

$$\pm 180^\circ = \sum_{i=0}^{0} \measuredangle (s-z_i) - \sum_{i=0}^{2} \measuredangle (s-p_i)$$

$$= \measuredangle (s+2) - \measuredangle (s-p_0) - \measuredangle (s+1) - \measuredangle (s)$$

$$= \varphi_0 - \theta_0 - \theta_1 - \theta_2$$

$$\cancel{-180^\circ} = 90^\circ - \theta_0 - \underbrace{\left(180^\circ - \tan^{-1}\left(\tfrac{2}{1}\right)\right)}_{\approx 117^\circ} - (\cancel{-}180^\circ - 45^\circ)$$



$117 - 90^\circ - 45^\circ = -\theta_0$

$\theta_0 = 18^\circ$

$$\frac{2}{l} = \tan 18^\circ \rightarrow l = \frac{2}{\tan 18^\circ}$$

$= 6.155 \longleftarrow l = -2 - p_0$

Let's pick $p_0 = -8$ $\longleftarrow$

$p_0 = -2 - 6.155 = -8.155$

④ utilize the magnitude Condition

$$\frac{\prod_{i=0} |s - z_i|}{\prod_{i=0}^{2} |s - p_i|} = \frac{1}{K_c}$$

$$K_c = \frac{M_{p_0} M_{p_1} M_{p_2}}{M_{z_0}} = \frac{\sqrt{6^2+2^2}\sqrt{1^2+2^2}\sqrt{2^2+2^2}}{2}$$

$$= 20$$

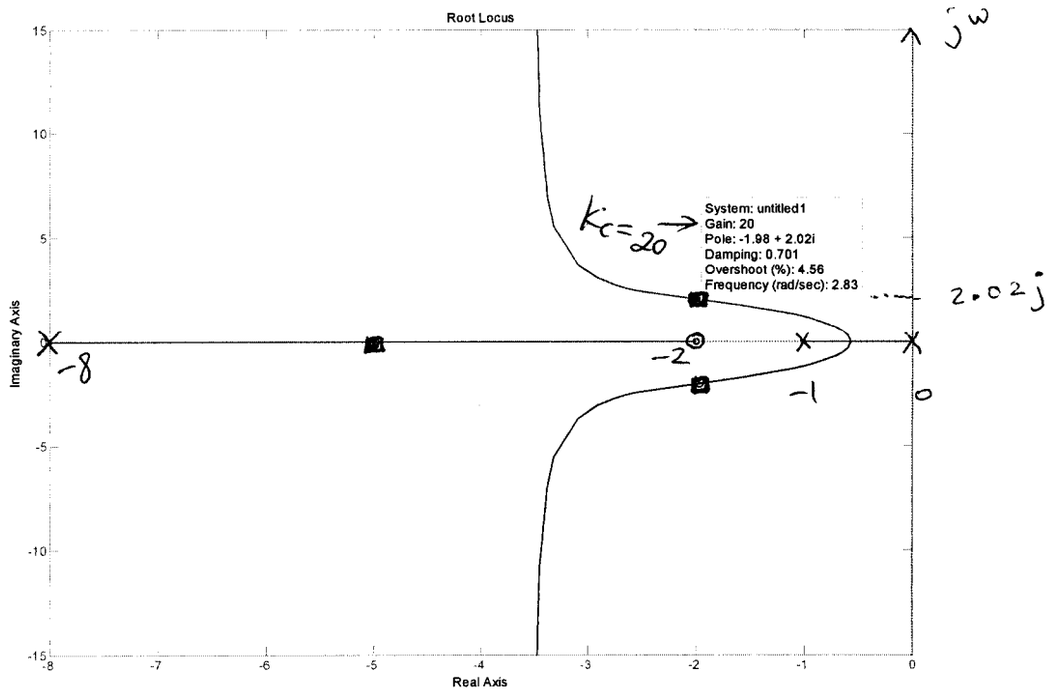
$$C(s) = 20 \cdot \frac{(s+2)}{(s+8)}$$

Root Locus of $1 + K_c \frac{s+2}{(s+8)(s+1)s} = 0$

Dominant poles at $s = -1.98 \pm 2.02j$ $\}$ $= 0.701$

rlocus(tf([1 2], [1 9 8 0]))

use matlab



■ closed loop poles at $K_c = 20$

X open loop poles

o open loop zero