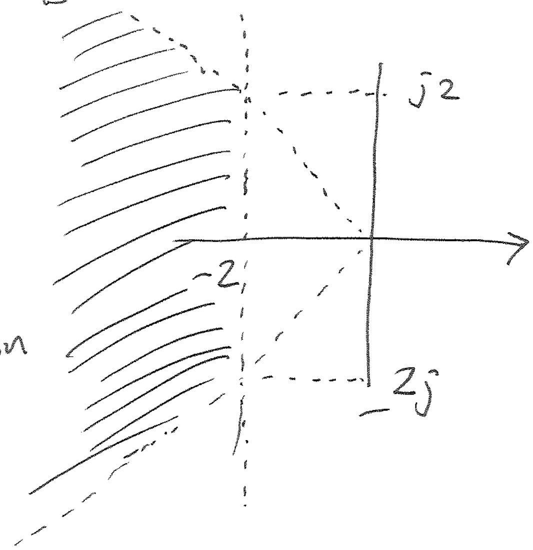
Satisfy A & B

$A \cap B$

$\Rightarrow$

good Region



Recall

$e^{j\omega t} \rightarrow \boxed{H(s)} \rightarrow H(j\omega)e^{j\omega t}$

Frequency response of systems

For given stable $G(s)$ with all poles $\in$ LHP

$$\mathcal{L}\Big\downarrow \begin{aligned} r(t) &= A \cos \omega_1 t \\ R(s) &= \frac{As}{s^2+\omega_1^2} \end{aligned}$$

$r \rightarrow \boxed{G} \rightarrow c$

$$C(s) = G(s)R(s) = \frac{A\ s}{(s-j\omega_1)(s+j\omega_1)} \cdot \frac{(s-z_1)(s-z_2)\cdots}{(s-p_1)(s-p_2)\cdots}$$

$$= \frac{k_1}{s-j\omega_1} + \frac{k_2}{s+j\omega_1} + \underbrace{\frac{k_3}{s-p_1} + \frac{k_4}{s-p_2}}_{} + \cdots\cdots$$

Re(all poles) < 0

$=$ all poles $\in$ LHP

$$\mathcal{L}^{-1}(C(s)) = k_1 e^{j\omega_1 t} + k_2 e^{-j\omega_1 t}$$

$$+ \underbrace{k_3 e^{p_1 t} + k_4 e^{p_2 t} + \cdots}_{\text{decay to zero}}$$

In steady state

$$c_{ss}(t) = k_1 e^{j\omega_1 t} + k_2 e^{-j\omega t}$$

only ~~forcing~~ so forced solution left.

Then cover up method of partial fraction expansion

$$K_1 = C(s)(s - j\omega_1)\Big|_{s=j\omega_1}$$

$$= G(s)\,\frac{As\,(s-j\omega_1)}{(s-j\omega_1)(s+j\omega_1)}\Bigg|_{s=j\omega_1}$$

$$= G(j\omega_1)\,\frac{A\,j\omega_1}{2j\omega_1} = G(j\omega_1)\,\frac{A}{2} = |G(j\omega_1)|\,e^{\phi j}\,\frac{A}{2}$$

likewise

$$K_2 = G(-j\omega_1)\,\frac{A}{2} = |G(j\omega_1)|\,e^{-\phi j}\,\frac{A}{2}$$

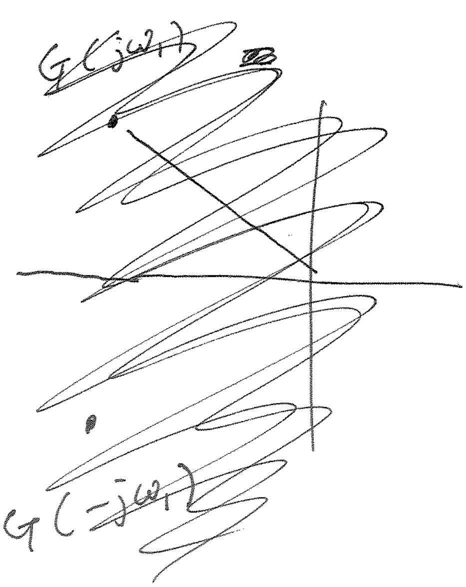
where $|G(j\omega_1)| = |G(-j\omega_1)|$

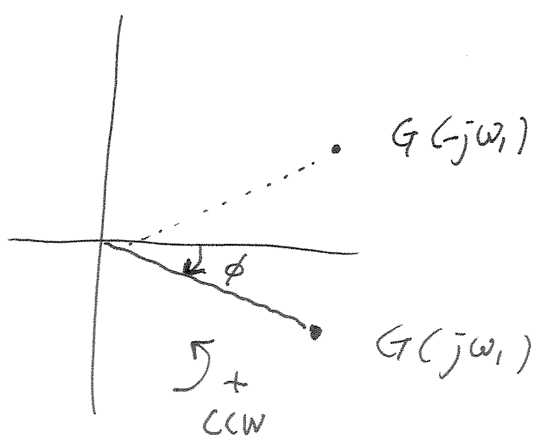
finally

$$C_{ss}(t) = \frac{A}{2}\left[\,G(j\omega_1)\,e^{j\omega_1 t} + G(-j\omega_1)\,e^{-j\omega_1 t}\right]$$

$$= A\,|G(j\omega_1)|\left[\frac{e^{j(\omega_1 t+\phi)} + e^{-j(\omega_1 t+\phi)}}{2}\right]$$

$$= A\,|G(j\omega_1)|\,\cos(\omega_1 t + \phi)$$





$\phi < 0$
phase delay

$$u(t) = e^{j\omega t} \longrightarrow \boxed{G(s)} \longrightarrow y_{ss}(t) = G(j\omega)e^{j\omega t}$$

output signal for $u(t) = e^{st}$ $\qquad$ $g(t) \rightarrow$ Impulse Response

$$\mathcal{L}^{-1}[Y(s)] = \mathcal{L}^{-1}[U(s)G(s)]$$

$$y(t) = u(t) * g(t) = \int_0^\infty g(\tau)\,u(t-\tau)\,d\tau$$

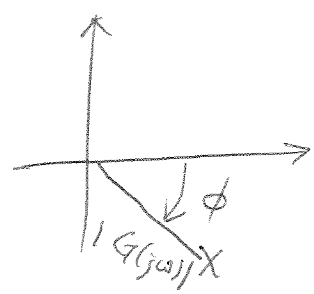
$$= \int_0^\infty g(\tau)\,e^{s(t-\tau)}\,d\tau$$

$$= \int_0^\infty g(\tau)\,e^{st}\,e^{-s\tau}\,d\tau$$

$$= \underbrace{\left[\int_0^\infty g(\tau)\,e^{-s\tau}\,d\tau\right]}_{=:\,\mathcal{L}(g) = G(s)} e^{st}$$

$$=: \underbrace{G(s)}_{TF}\,e^{st}$$

$$u(t) = e^{st} \longrightarrow \boxed{G(s)} \longrightarrow y_{ss}(t) = G(s)\,e^{st}$$



$$u(t) = \cos\omega t + j\sin\omega t \longrightarrow \boxed{G(s)} \longrightarrow y_{ss}(t)$$

$$= G(j\omega)\,e^{j\omega t}$$

$$= |G(j\omega)|\,e^{j\phi}\cdot e^{j\omega t}$$

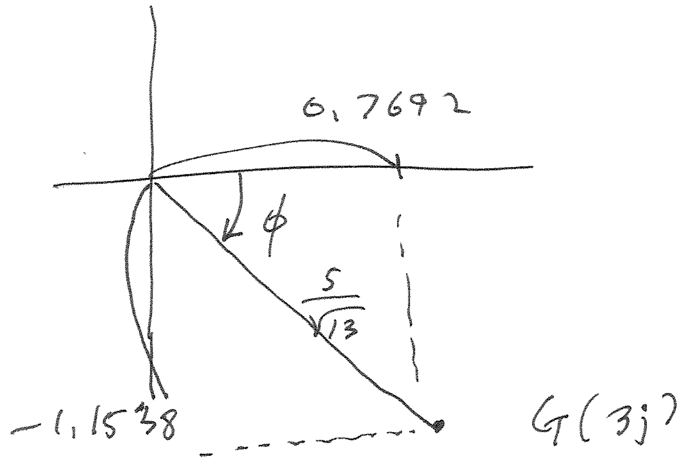
$$= |G(j\omega)|\,e^{j(\omega t+\phi)}$$

$$= |G(j\omega)|\left[\cos(\omega t+\phi) + j\sin(\omega t+\phi)\right]$$

ex) $G(s) = \dfrac{5}{s+2}$ input $= \cos 3t$

step1) $G(s)\Big|_{s=j3} = \dfrac{5}{3j+2} = \dfrac{5(2-3j)}{3^2+2^2}$

$$= \frac{5}{13}(2-3j)$$



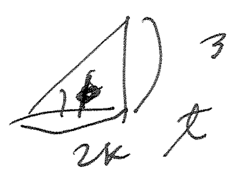
$$= 0.7692 - 1.1538j$$

$$\phi = -\tan^{-1}\left(\frac{3}{2}\right)$$

$$= -\mathrm{atan}\left(\frac{3}{2}\right)$$

$$= -0.9828 \text{ (radian)}$$

$$G(3j) = re^{\phi j} = \frac{5}{\sqrt{13}} \times e^{-0.9828j}$$



$\tan x = \dfrac{\sin x}{\cos x}$

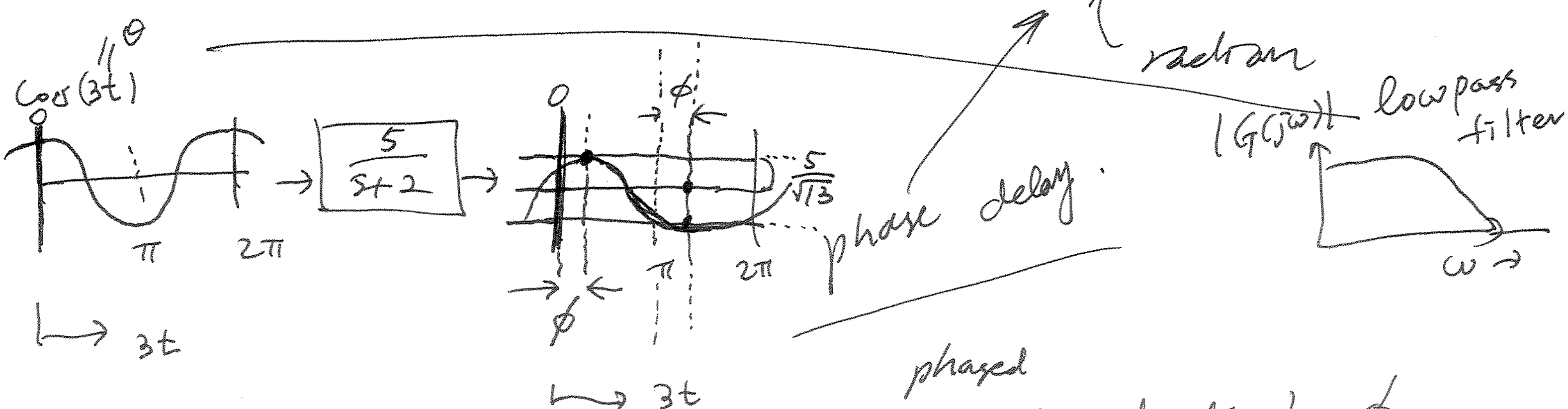
answer

$$= -0.9828\left(\frac{360^\circ}{2\pi}\right)$$

$$= -56.3^\circ$$

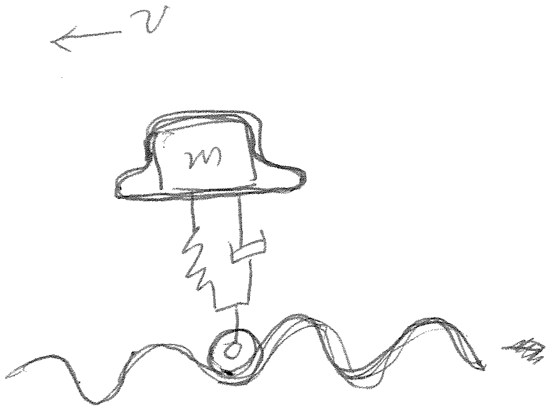
답. $c_{ss}(t) = \dfrac{5}{\sqrt{13}} \cos(3t - 0.9828)$

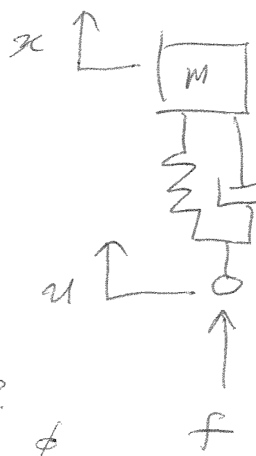
radian



$\ast$ phased delayed output by ϕ

→ dynamical system 은 has memory

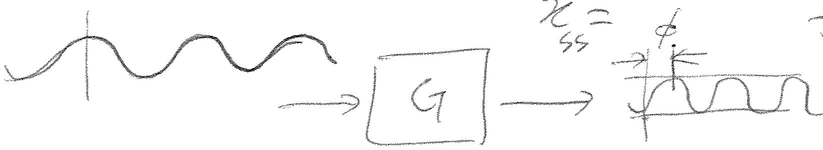




$$\frac{X(s)}{U(s)} = \frac{bs+k}{ms^2+bs+k}$$

$$\frac{X(s)}{U(s)} = \frac{\frac{b}{m}s+\frac{k}{m}}{s^2+\frac{b}{m}s+\frac{k}{m}}$$

$u = \cos \omega t$



$$u = e^{j\omega t} \rightarrow \quad \rightarrow y_{ss} = G(j\omega)\, e^{j\omega t} = |G(j\omega)|\, e^{\phi}\, e^{j\omega t}$$

$$G(s) = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$x_{ss}(t) = |G(j\omega)| \cos(\omega t + \phi)$$

$$X(s) = G(s)\cdot U(s) = \left[\frac{2\zeta\omega_n s}{s^2+2\zeta\omega_n s+\omega_n^2} + \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}\right]\frac{1}{s}$$

$$= \frac{2\zeta\omega_n}{s^2+2\zeta\omega_n s+\omega_n^2} + \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}\,\frac{1}{s}$$

$$\left(G_2(s) := \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}\right) = \frac{2\zeta}{\omega_n}\cdot\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} + \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}\cdot\frac{1}{s}$$

$$\frac{1}{s} \longrightarrow \boxed{G_2} \longrightarrow Y \qquad = \frac{2\zeta}{\omega_n}\cdot G_2(s) + G_2(s)\,\frac{1}{s}$$

$$x(t) = \mathcal{L}^{-1}[G(s)\cdot U(s)] = \frac{2\zeta}{\omega_n}\cdot\mathcal{L}^{-1}[G_2(s)] + \mathcal{L}^{-1}\left[G_2(s)\frac{1}{s}\right]$$

$$x(t) = \frac{2\zeta}{\omega_n}\cdot y_{im}(t) + y_{step}(t)$$