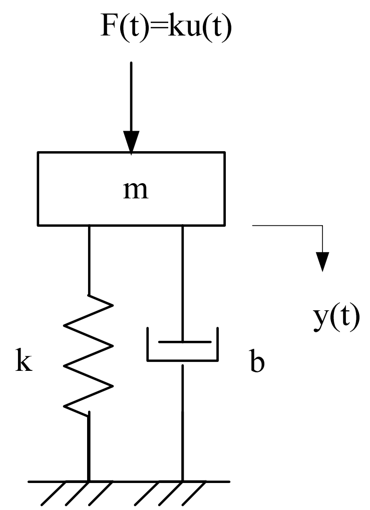
Second order system (mass-spring-damper system)

- ODE : $$\ddot{y}(t) + \frac{b}{m}\dot{y}(t) + \frac{k}{m}y(t) = \frac{k}{m}u(t)$$



ζ : dampling ratio, ω_n : natural frequency

$$2\zeta\omega_n = b/m,\ \omega_n^2 = k/m$$

$$\ddot{y}(t) + 2\zeta\omega_n\dot{y}(t) + \omega_n^2 y(t) = \omega_n^2 u(t)$$

- Transfer function :

$$\frac{Y(s)}{U(s)} = H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n^2 s + \omega_n^2}$$

Polar vs. Cartesian representations.

Cartesian representation :

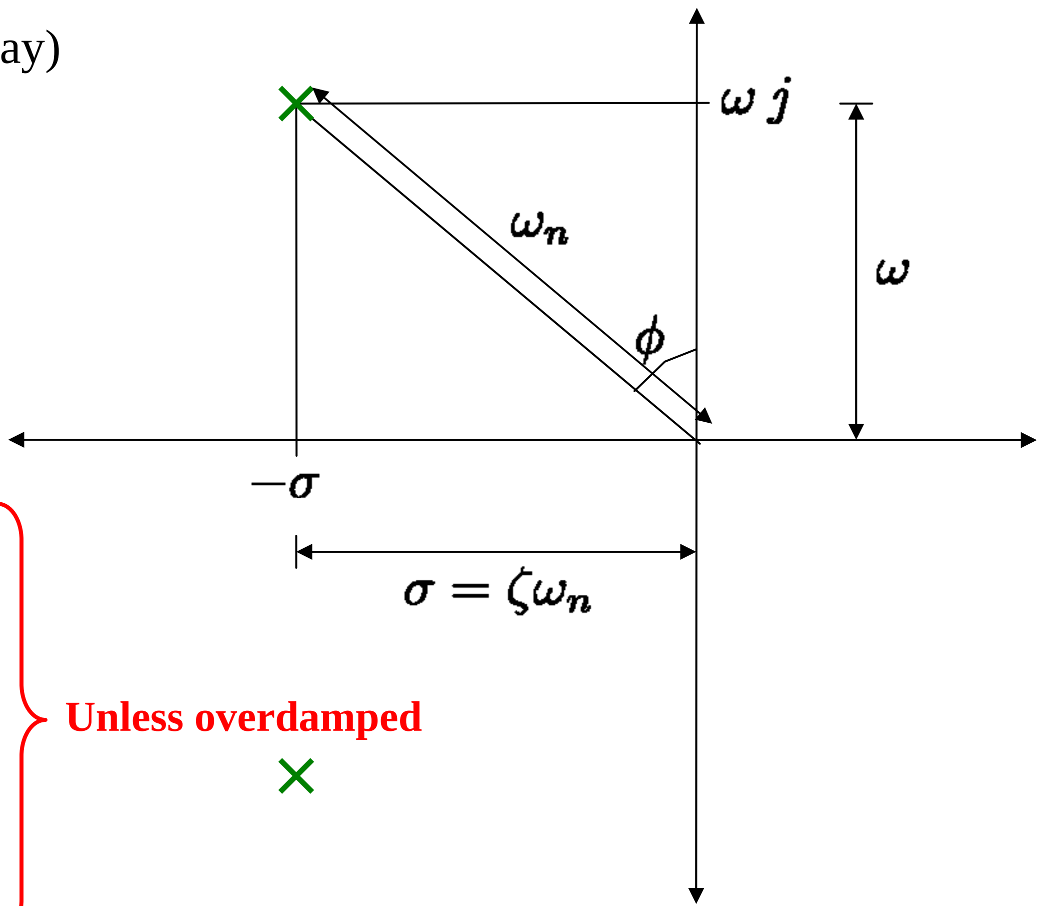
ω … Imaginary part (frequency)

σ … Real part (rate of decay)

Polar representation :

ω_n … natural frequency

ζ … damping ratio



$$\omega = |Im(p)|$$
$$\sigma = |Re(p)|$$
$$\omega_n = |p| = \sqrt{\sigma^2 + \omega^2}$$
$$\zeta = \sin(\phi) = \sigma/\omega_n$$
$$\omega = \omega_n\sqrt{1 - \zeta^2}$$

Unless overdamped