1. A company has decided to air condition their manufacturing facility. They are trying to decide between an absorption refrigeration system and a vapor compression refrigeration system. The following economic data is available:

<table>
<thead>
<tr>
<th></th>
<th>Absorption System</th>
<th>Vapor Compression System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital Cost</td>
<td>$125,000</td>
<td>$85,000</td>
</tr>
<tr>
<td>Annual Operating Cost</td>
<td>$15,000</td>
<td>$20,000</td>
</tr>
</tbody>
</table>

The life of both systems is estimated to be 15 years and the interest rate for the company is 13%. Using a present value analysis determine which system should be purchased. What would be the annual operating cost of the vapor compression system needed to make the two systems equivalent in terms of cost?

**Solution:**

Our present value for either system will be calculated from

\[ PV = P_{\text{cap}} + A_{\text{op}} \cdot \frac{P}{A}(i, N) \]

where our P/A function is given by

\[ \frac{P}{A} = \frac{(1+i)^N - 1}{i(1+i)^N} = \frac{(1 + 0.13)^{15} - 1}{0.13(1 + 0.13)^{15}} = 6.4624 \]

Then

\[ (PV)_A = 125,000 + 15,000 \cdot 6.4624 = 221,936 \]

and

\[ (PV)_V = 85,000 + 20,000 \cdot 6.4624 = 214,248 \]

So the vapor compression system has the lower cost and should be purchased. Since electricity prices have a tendency to increase over time, it might be of interest to determine an operating cost for the vapor compression system that would make its cost equivalent to that of the absorption system. Hence we want

\[ (PV)_V = (PV)_A = 221,936 \]
so that

\[
(A_{op})_V = \frac{(PV)_A - (P_{cap})_V}{P/A(i,N)} = \frac{221,936 - 85,000}{6.4624} = $21,890
\]

or a 6% increase.

2. A homeowner is considering replacing their current gas fuel heating system with a solar heating system. Both systems are to last 20 years and the homeowner’s interest rate is 8%. The initial cost for the furnaces is $1200 and the annual fuel cost $600. Determine the capital cost of the solar system that will make it competitive with the gas fueled system.

**Solution:**
To determine the acceptable capital cost of our solar system, we will let it be equal to the present value cost of the gas fueled system or

\[
(P_{cap})_S = (PV)_G = (P_{cap})_G + (A_{op})_G \cdot \frac{P}{A(i,N)}
\]

For \( i = 0.08 \) and \( N = 20 \) years, we find

\[
P = \frac{(1+i)^N - 1}{i(1+i)^N} = \frac{(1+0.08)^{20} - 1}{0.08(1+0.08)^{20}} = 9.8181
\]

so that

\[
(P_{cap})_S = 1200 + 600 \cdot 9.8181 = $7091
\]

3. An experimental research station located in the wilderness must provide its own electricity. Two options are being considered: wind turbine and gasoline powered generator. Determine the energy cost in $/(kW·hr) for each proposed system. The following conditions apply:

<table>
<thead>
<tr>
<th>System</th>
<th>Generating Power (kW)</th>
<th>% Up Time</th>
<th>Capital Cost</th>
<th>Annual Maintenance Cost</th>
<th>Annual Operating Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind Turbine</td>
<td>15</td>
<td>50%</td>
<td>$15,000</td>
<td>$500</td>
<td>$0</td>
</tr>
<tr>
<td>Gasoline Generator</td>
<td>10</td>
<td>75%</td>
<td>$500</td>
<td>$500</td>
<td>$2,500</td>
</tr>
</tbody>
</table>

For the calculation an interest of rate of 15% and a lifetime of 20 years should be used.
Solution:
For the wind turbine we have:
\[
A = P_{\text{cap}} (A/P,0.15,20) + A_m + A_o
\]
\[
(A/P,i,N) = \frac{i(1+i)^N}{(1+i)^N - 1} = \frac{(0.15)(1.15)^{20}}{(1.15)^{20} - 1} = 0.1598
\]
\[
A = (15,000)(0.1598) + 500 + 0 = $2896.42
\]
\[
E = \text{(power)(up time)(hrs/year)} = (15)(0.5)(24)(365) = 65,700 \text{ kW·hr}
\]
\[
cost = \frac{A}{E} = \frac{2896.42}{65,700} = 0.0441 \$/\text{(kW·hr)}
\]

For the gasoline generator we have:
\[
A = P_{\text{cap}} (A/P,0.15,20) + A_m + A_o
\]
\[
A = (500)(0.1598) + 500 + 2500 = $3079.88
\]
\[
E = \text{(power)(up time)(hrs/year)} = (10)(0.75)(24)(365) = 65,700 \text{ kW·hr}
\]
\[
cost = \frac{A}{E} = \frac{3079.88}{65,700} = 0.0469 \$/\text{(kW·hr)}
\]

Based on these numbers we would choose the wind turbine system.

4. A solar collector receives heat transfer from the sun at a rate of 0.315 kW per m² of collector area and provides energy to a storage unit whose temperature remains constant at 500 K. A heat engine interacts with the storage unit as its high temperature reservoir, generates 1000 kW of work output, and interacts with the surroundings at 20°C as its low temperature reservoir. Determine the minimum theoretical collector area required.

Solution:
We begin by sketching our device interactions
The heat transfer from the storage unit is provided by the solar collector and is given by
$$\dot{Q}_H = (0.315)A_{\text{collector}}$$
Since we are looking for the minimum collector area, we will assume that the heat engine operates as a Carnot cycle, then
$$\eta_{\text{th}} = \eta_{\text{Carnot}} = 1 - \frac{T_L}{T_H} = 1 - \frac{(20 + 273)}{500} = 0.414$$
But by definition we have
$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_H}$$
or solving for the heat transfer
$$\dot{Q}_H = \frac{\dot{W}_{\text{net}}}{\eta_{\text{th}}} = \frac{1000}{0.414} = 2415.5 \text{ kW}$$
and solving for the collector area
$$A_{\text{collector}} = \frac{\dot{Q}_H}{(0.315)(0.315)} = \frac{2415.4}{(0.315)(0.315)} = 7668 \text{ m}^2$$

5. In the month of January, a certain dwelling requires $2.4 \times 10^6$ kJ per day to maintain its temperature at $20^\circ C$. As a means of proving the required heating, a heat pump cycle is under consideration that would operate between the dwelling and a nearby pond at $10^\circ C$. The manufacturer claims that the net work required to operate the heat pump is $1.6 \times 10^5$ kJ per day. Evaluate this claim.

**Solution:**
Again we begin by sketching our device interactions
To evaluate the claim we must compare the operating COP to the Carnot cycle COP for the heat pump:

\[
\text{COP}_{\text{Carnot}} = \frac{1}{1 - \frac{T_L}{T_H}} = \frac{1}{1 - \frac{283}{293}} = 29.3
\]

The operating COP is given by

\[
\text{COP}_{\text{actual}} = \frac{\dot{Q}_H}{W_{\text{net}}} = \frac{2.4 \times 10^6}{1.5 \times 10^5} = 15
\]

and since \( \text{COP}_{\text{actual}} < \text{COP}_{\text{Carnot}} \), the claim is possible.

6. An innovative way of power generation involves the utilization of geothermal energy, the energy of hot water that exists naturally underground (hot springs), as the heat source. If a supply of hot water at 140°C is discovered at a location where the environmental temperature is 20°C, determine the maximum thermal efficiency a geothermal plant built at that location can have. If the power output of the plant is to be 5 MW, what is the minimum mass flow rate of hot water needed?

**Solution:**

We begin by sketching our device interactions.
The maximum thermal efficiency will occur when the heat engine operates as a Carnot cycle, 
\[ \eta_{th} = \eta_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{(20 + 273)}{(140 + 273)} = 0.291 \]

The minimum mass flow rate of hot water corresponds to the maximum thermal efficiency or 
\[ (Q_H)_{min} = \frac{W_{net}}{\eta_{Carnot}} = \frac{5000}{0.291} = 17,208 \text{ kW} \]

Performing a first law analysis on the hot water stream we have 
\[ Q = \dot{m}(h_{out} - h_{in}) \]

For the minimum flow rate we will assume that the hot water is cooled down to the environment temperature, then 
\[ \dot{m} = \frac{Q}{c_p(T_{out} - T_{in})} = \frac{-17,208}{(4.1978)(20 - 140)} = 34.2 \text{ kg/s} \]

7. A heat pump provides 30,000 Btu/hr to maintain a dwelling at 68°F on a day when the outside temperature is 35°F. The power input to the pump is 1 hp. If electricity costs 8 cents per kilowatt-hour, compare the actual operating cost per day with the minimum theoretical operating cost per day.
Solution:
We sketch our device interactions

The cost is given by

$$\text{Cost} = (0.08)W_{\text{net}}$$

For the actual cost we have

$$(\text{Cost})_{\text{act}} = (0.08)(1)(0.7457 \text{kW/hp})(24 \text{ hr/day})$$

$$= $1.43$$

To calculate the minimum cost we will allow the heat pump to operate as a Carnot cycle, so that

$$\text{COP}_{\text{Carnot}} = \frac{1}{\frac{T_L}{T_H}} = \frac{1}{\frac{495}{528}} = 16$$

Then the minimum possible power input is

$$\left(\dot{W}_{\text{net}}\right)_{\text{min}} = \frac{\dot{Q}_H}{\text{COP}_{\text{Carnot}}} = \frac{30,000}{16} = 1875 \text{ Btu/hr}$$

$$= 0.5495 \text{ kW}$$

and the minimum cost is

$$(\text{Cost})_{\text{min}} = (0.08)(0.5495)(24 \text{ hr/day})$$

$$= $1.06$$