Basic Thermal Engineering

Steady state energy balance

Work Devices: \( \dot{W}_{\text{in}} = \dot{m}\left(\overline{h}_2 - \overline{h}_1 + \frac{(\overline{v}_2^2 - \overline{v}_1^2)}{2} + g(z_2 - z_1)\right) \)

Heat Devices: \( \dot{Q}_{\text{in}} = \dot{m}\left(\overline{h}_2 - \overline{h}_1 + \frac{(\overline{v}_2^2 - \overline{v}_1^2)}{2} + g(z_2 - z_1)\right) \)

Heat Exchanges: \( \dot{Q}_{\text{in}} = \dot{m}\left(\overline{h}_2 - \overline{h}_1 + \frac{(\overline{v}_2^2 - \overline{v}_1^2)}{2} + g(z_2 - z_1)\right) \)

Ideal Performance

Work Devices: \( \Delta s = 0 \)
Heat Devices: \( \Delta P = 0 \)

Device Efficiencies

Work Consuming Devices: \( \dot{W}_{\text{act}} = \frac{\dot{W}_{\text{ideal}}}{\eta_s} \)

Work Producing Devices: \( \dot{W}_{\text{act}} = \eta_s \dot{W}_{\text{ideal}} \)

Property Evaluation

Ideal Gas: \( \Delta h = c_p \Delta T, \Delta s = c_p \ln(T_2/T_1) - R \ln(P_2/P_1) \)
Incompressible Liquid: \( \Delta h = c_p \Delta T + \nu \Delta P, \Delta s = c_p \ln(T_2/T_1) \)
Compressible Substance: Must use property tables/

Time Value of Money Functions

Definitions

A: Annual Cost or Value
P: Present Cost or Value
F: Future Cost or Value
N: number of compound periods
i: interest rate in decimal form.

Conversion Factors

\( F = P \cdot (F/P, i, N) = (1 + i)^N \)
\( F = A \cdot (F/A, i, N) = \frac{(1 + i)^N - 1}{i} \)
\( P = F \cdot (P/F, i, N) = \frac{1}{(1 + i)^N} \)
\( P = A \cdot (P/A, i, N) = \frac{(1 + i)^N - 1}{i(1 + i)^N} \)
\( A = F \cdot (A/F, i, N) = \frac{i}{(1 + i)^N - 1} \)
$A = P \cdot (A/P, i, N)$ where $(A/P, i, N) = \frac{i(1+i)^N}{(1+i)^N - 1}$

**Ocean Energy Calculations**

**Wave Energy Conversion**

Assuming that the energy of a wave is in its potential energy, we can write the wave power as

$$\dot{W}_{\text{wave}} = \rho_{sw} g (\Delta z)_{\text{wave}} V_{\text{wave}} f_{\text{wave}}$$

where

$\dot{W}_{\text{wave}}$: power of the wave

$\rho_{sw}$: density of the sea water

$g$: acceleration due to gravity

$(\Delta z)_{\text{wave}}$: height of wave

$V_{\text{wave}}$: volume of wave

$f_{\text{wave}}$: frequency of wave

Assuming the wave is sinusoidal, and approximating the sine function as a triangle, we could write

$$V_{\text{wave}} = \frac{1}{2} (\Delta z)_{\text{wave}} t_{\text{wave}} w_{\text{wave}}$$

where

$t_{\text{wave}}$: thickness of the wave

$w_{\text{wave}}$: width of the wave

**Oscillating Water Column**

We consider that we have an initial state of air in the chamber (state 1), a final state of the air in the chamber (state 2), the state of the air entering the turbine (state 3), and the turbine exit state (state 4). We assume that the following are specified in the problem

$P_1, V_1, T_1, V_2$, and $P_4$

and the following properties of air are known

$c_p$, $c_v$, and $k$

The polytropic coefficient for the compression of the air column can be taken to be 1.2. The power produced a two chamber OWC is given by

$$\dot{W}_{\text{OWC}} = 2W_{\text{turb}} \cdot f_{\text{wave}}$$

The turbine work is given by

$$W_{\text{turb}} = m_3 C_p (T_3 - T_4)$$

with

$$T_4 = \frac{T_1 + T_2}{2} \left( \frac{P_4}{(P_1 + P_2)/2} \right)^{\frac{k-1}{k}}$$

$$T_3 = \frac{T_1 + T_2}{2}$$

$$m_3 = m_1 - m_2$$

$$m_2 = P_2 V_2/(RT_2)$$
\[ P_2 = P_1 \left( \frac{V_1}{V_2} \right)^n \]

All of these calculations require a value of \( T_2 \), which is given by
\[
T_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}
\]

where
\[
a = (m_1C_p/2) \\
b = [P_2V_2C_v/R - m_1C_vT_1 + m_1C_pT_1/2 - P_2V_2C_p/(2R) + (P_2 - P_1V_1)/(1-n)] \\
c = -P_2V_2C_pT_1/(2R)
\]

**Hydraulic Pump Float System**

The power of a float system can be calculated by
\[
\hat{W}_{\text{float}} = \eta_{\text{float}} m_{\text{float}} g(\Delta z)_{\text{wave}} f_{\text{wave}} \text{ in } W
\]

where
\[
\eta_{\text{float}}: \text{conversion efficiency of the float WEC system} \\
m_{\text{float}}: \text{mass of the float}
\]

**Tide Energy Conversion (TEC)**

Assuming that water only flows through the turbine of a tidal energy system from low tide to neutral tide (the point where the water level in the reservoir is equal to the ocean level), the power output of a TEC system can then be calculated with
\[
\hat{W}_{\text{TEC}} = \frac{\rho_{sw}(z_{ht} - z_{nt})A_{\text{res}}\eta_{\text{turb}}w_{\text{ideal}}}{\tau}
\]

where
\[
z_{ht}: \text{height of high tide} \\
z_{nt}: \text{height of neutral tide} \\
A_{\text{res}}: \text{surface area of the reservoir} \\
\tau: \text{tide period} \\
\eta_{\text{turb}}: \text{efficiency of the hydraulic turbine} \\
w_{\text{ideal}}: \text{specific ideal work of the tide}
\]

It can be shown that
\[
w_{\text{ideal}} = \frac{3}{4} g\Delta z
\]

**Current Energy Power System**

The current power density is defined by
\[
\hat{W}_{\text{current}}^\prime = \frac{1}{2} \rho_{sw} \vec{v}_{\text{current}}^3
\]

The power output of a marine turbine would be given by
\[
\hat{W}_{\text{mturb}} = \eta_{\text{mturb}} A\hat{W}_{\text{current}}^\prime
\]

where
\[
\eta_{\text{mturb}}: \text{marine turbine efficiency}
\]
A: current flow area that is captured by the marine turbine. For a marine turbine with rotor diameter, D

\[ A = \pi D^2/4 \]

**Ocean Thermal Energy Conversion (OTEC)**

The maximum efficiency or work output that can be achieved by using the Carnot cycle efficiency is given by

\[
\eta_{\text{max}} = 1 - \frac{T_L}{T_H}
\]

The minimum amount of hot water consumed by an OTEC system would be given by

\[
\dot{m}_H = \frac{\dot{W}_{\text{OTEC}}}{\eta_{\text{max}} c_{p,H}(\Delta T)_H}
\]

where

\( \dot{W}_{\text{OTEC}} \): required power output of the OTEC system

\( c_{p,H} \): specific heat of the hot sea water

\( (\Delta T)_H \): allowed temperature change for the hot sea water

For a Rankine cycle based system, the condenser and boiler temperatures are given by

\[
T_{\text{boiler}} = T_{H,\text{in}} - \frac{(\Delta T)_H}{\varepsilon_B}
\]

\[
T_{\text{cond}} = T_{C,\text{in}} + \frac{(\Delta T)_C}{\varepsilon_C}
\]

where \( \varepsilon \) is the heat exchanger effectiveness of the boiler or condenser.

**Geothermal Calculations**

**Hot Geothermal Sources**

Modeling as a heat engine, we have
The high temperature reservoir is the hot geothermal source and the low temperature heat reservoir is typically the ambient air. Our appropriate equations are:

\[ Q_H = W_{net} + Q_L \]
\[ \eta_{th} = \frac{W_{net}}{Q_H} \]
\[ \eta_{Carnot} = 1 - \frac{T_L}{T_H} \]

An energy on the geothermal source would give

\[ \dot{Q}_H = \dot{m}_{GS} (\hat{h}_\text{in} - \hat{h}_\text{out}) \]

where \( \dot{m}_{GS} \) is the mass flow rate of the hot water or steam from the geothermal source. For a hot water source, we can write

\[ \dot{Q}_H = \dot{m}_{GS} c_{P, hw} (T_\text{in} - T_\text{out}) \]

**Cold Geothermal Source**

For most locations in the U.S. we may take the ground temperature 10 feet below the surface to be 14°C. For heating purposes we may use a heat pump system. An interaction diagram is shown below.
To calculate the performance of the various devices, we can use

\[ \dot{Q}_H = \dot{m}_{ref} (\hat{h}_{in} - \hat{h}_{out})_{\text{cond}} \]
\[ \dot{Q}_L = \dot{m}_{ref} (\hat{h}_{out} - \hat{h}_{in})_{\text{evap}} \]
\[ \dot{W}_{\text{net}} = \dot{m}_{ref} (\hat{h}_{out} - \hat{h}_{in})_{\text{comp}} \]

To estimate the earth required for either underground heating of cooling we can use the following equation.

\[ m_{\text{earth}} c_{P,\text{earth}} (\Delta T)_{\text{earth}} = \dot{Q}_{\text{HE}} \tau \]