ME 417
Design of Alternative Energy Systems

Geothermal Problem Solutions

1. A hot geothermal source has a hot water at 65°C with a flow rate of 0.05 kg/s. Determine the flow rate of R-134a that can be boiled by this source.

**Solution:**
An energy balance on the boiler gives
\[ \dot{m}_{R134a} h_{fg,R134a} = \dot{m}_{w} c_{P,w} (T_{w,in} - T_{w,out}) \]
In order to use this equation, we must determine the operating temperature of the boiler. With the hot water entering at 65°C, we will choose 60°C as the boiler temperature. Then going to the sR-134a table we have
\[ h_{fg,R134a} = 113.52 \text{ kJ/kg} \] and \[ T_{w,out} = 60°C \].
Then
\[ \dot{m}_{R134a} = \dot{m}_{w} c_{P,w} (T_{w,in} - T_{w,out})/h_{fg,R134a} \]
\[ = (0.05)(4.2)(65 - 60)/(113.52) = 0.00925 \text{ kg/s} \]

2. A vapor compression refrigeration system operating between –8°C and 30°C is to be replaced with an underground cooling loop. Both systems are to supply a cooling load of 2 kW and 75 cooling days. Determine the break even capital cost for the underground cooling loop for an interest rate of 12%, a lifetime of 25 years, and an electricity cost of 0.07 $/kWh.

**Solution:**
Ignoring the annual cost of the pump for the underground cooling loop, the differential capital cost between the two systems will be due to the electricity consumed by the vapor compression refrigeration system, or
\[ \Delta P_{\text{cap}} = \dot{W}_{\text{comp}} \times \text{(hours of operation per year)} \times \text{(energy unit cost)} \times (P/A,i,N) \]
where
\[ \text{(hours of operation per year)} = (75)(24) = 1800 \text{ hr/yr} \]
\[ \text{(energy unit cost)} = 0.07 \text{ $/kWh} \]
\[ (P/A,i,N) = \frac{(1+i)^N - 1}{i(1+i)^N} = \frac{(1 + 0.12)^{25} - 1}{(0.12)(1.12)^{25}} = 7.843 \]
\[ \dot{W}_{\text{comp}} = \dot{m}_{R134a} (\Delta h)_{\text{comp}} \]
\[ \dot{m}_{\text{R134a}} = \frac{\text{cooling load}}{(\Delta h)_{\text{evap}}} \]

For the power of the compressor and the mass flow rate of R-134a, we will need to perform an analysis on the refrigeration cycle. We begin with the entrance to the compressor, where we will have saturated vapor at \(-8^\circ\text{C}\). Calling this state 1, we can go to the R-134a tables and find

\[ h_1 = 242.54 \text{ kJ/kg} \quad s_1 = 0.9239 \text{ kJ/(kg \cdot K)} \quad P_1 = 217 \text{ kPa} \]

Across the compressor, we have an isentropic process, so that

\[ s_2 = s_1 = 0.9239 \text{ kJ/(kg \cdot K)} \]

From the R-134a tables we find

\[ P_2 = 0.77 \text{ MPa} \quad h_2 = 273 \text{ kJ/kg} \]

Across the evaporator we go from a two phase mixture to a saturated vapor. The two phase mixture will have the same enthalpy as the entrance to the valve, which is saturated liquid at \(-8^\circ\text{C}\). Then

\[ h_4 = 39.54 \text{ kJ/kg} \]

and

\[ (\Delta h)_{\text{evap}} = 242.54 - 39.54 = 203 \text{ kJ/kg} \]

Then

\[ \dot{m}_{\text{R134a}} = \frac{2}{203} = 0.00985 \text{ kg/s} \]

\[ \dot{W}_{\text{comp}} = (0.00985)(273 - 242.54) = 0.30 \text{ kW} \]

Finally

\[ \Delta P_{\text{cap}} = (0.30)(1800)(0.07)(7.843) = 297 \]

3. Consider an underground heat pump that serves a building with a heating load of 6.5 kW. Determine the volume of earth (in cubic yards) that must participate in the heating for a cooling period of 150 days and an allowable ground temperature drop of 15°C. The properties of the earth are estimated to be

\[ c_p = 0.8 \text{ kJ/(kg \cdot K)} \]

\[ \rho = 1520 \text{ kg/m}^3 \]

**Solution:**

Our equation is

\[ m_{\text{earth}} c_p,\text{earth} (\Delta T)_{\text{earth}} = \dot{Q}_{\text{HE}} \tau \]

Solving for the mass of earth

\[ m_{\text{earth}} = \frac{\dot{Q}_{\text{HE}} \tau}{c_p,\text{earth} (\Delta T)_{\text{earth}}} = \frac{(6.5)(150)(24)(3600)}{(0.8)(15)} = 7.02 \times 10^6 \text{ kg} \]

The volume is then given by
\[ V_{\text{earth}} = \frac{m_{\text{earth}}}{\rho} = \frac{(7.02 \times 10^{-6})}{(1520)} = 4618 \text{ m}^3 \]
or about 6024 cubic yards.