Hot Geothermal Sources
Modeling as a heat engine, we have

\[
Q_H = W_{\text{net}} + Q_L
\]

\[
\eta_{th} = \frac{W_{\text{net}}}{Q_H}
\]

\[
\eta_{\text{Carnot}} = 1 - \frac{T_L}{T_H}
\]
An energy on the geothermal source would give
\[ \dot{Q}_H = \dot{m}_{GS}(\hat{h}_{in} - \hat{h}_{out}) \]
where \( \dot{m}_{GS} \) is the mass flow rate of the hot water or steam from the geothermal source. For a hot water source, we can write
\[ \dot{Q}_H = \dot{m}_{GS}c_{P,\text{hw}}(T_{in} - T_{out}) \]

**Cold Geothermal Source**
For most locations in the U.S. we may take the ground temperature 10 feet below the surface to be 14ºC. For heating purposes we may use a heat pump system. An interaction diagram is shown below.

![Interaction Diagram](image)

The high temperature reservoir is the building being heated and the low temperature heat reservoir is the earth. Our appropriate equations are:
\[ Q_H = W_{\text{net}} + Q_L \]
\[ (COP)_{HP} = \frac{Q_H}{W_{\text{net}}} \]
\[ COP_{\text{Carnot}} = \frac{1}{1 - \frac{T_L}{T_H}} \]

The plant used for the heat pump is typically based on the vapor compression system shown below.
To calculate the performance of the various devices, we can use

\[ \dot{Q}_H = \dot{m}_{ref} (\hat{h}_{in} - \hat{h}_{out})_{cond} \]

\[ \dot{Q}_L = \dot{m}_{ref} (\hat{h}_{out} - \hat{h}_{in})_{evap} \]

\[ \dot{W}_{net} = \dot{m}_{ref} (\hat{h}_{out} - \hat{h}_{in})_{comp} \]

To estimate the earth required for either underground heating or cooling we can use the following equation.

\[ m_{earth} c_{P,earth} (\Delta T)_{earth} = \dot{Q}_{HE} \tau \]