ME 417
Design of Alternative Energy Systems

Sample Equation Sheet

**Basic Thermal Engineering**
Steady state energy balance

- **Work Devices:** \[ \dot{W}_{in} = m \left\{ (h_2 - h_1) + (\bar{v}_2^2 - \bar{v}_1^2) / 2 + g(z_2 - z_1) \right\} \]
- **Heat Devices:** \[ \dot{Q}_{in} = m \left\{ (h_2 - h_1) + (\bar{v}_2^2 - \bar{v}_1^2) / 2 + g(z_2 - z_1) \right\} \]
- **Heat Exchanges:** \[ \dot{Q}_{in} = m \left\{ (h_2 - h_1) + (\bar{v}_2^2 - \bar{v}_1^2) / 2 + g(z_2 - z_1) \right\} \]

**Ideal Performance**
- **Work Devices:** \( \Delta s = 0 \)
- **Heat Devices:** \( \Delta P = 0 \)

**Device Efficiencies**
- **Work Consuming Devices:** \[ \dot{W}_{act} = \frac{\dot{W}_{ideal}}{\eta_s} \]
- **Work Producing Devices:** \[ \dot{W}_{act} = \eta_s \dot{W}_{ideal} \]

**Property Evaluation**
- **Ideal Gas:** \( \Delta h = c_p \Delta T, \Delta s = c_p \ln(T_2/T_1) - R \ln(P_2/P_1) \)
- **Incompressible Liquid:** \( \Delta h = c_p \Delta T + v \Delta P, \Delta s = c_p \ln(T_2/T_1) \)
- **Compressible Substance:** Must use property tables/

**Time Value of Money Functions**

**Definitions**
- **A:** Annual Cost or Value
- **P:** Present Cost or Value
- **F:** Future Cost or Value
- **N:** number of compound periods
- **i:** interest rate in decimal form.

**Conversion Factors**
- \( F = P \cdot (F/P,i,N) = (1 + i)^N \)
- \( F = A \cdot (F/A,i,N) = \frac{(1+i)^N - 1}{i} \)
- \( P = F \cdot (P/F,i,N) = \frac{1}{(1+i)^N} \)
- \( P = A \cdot (P/A,i,N) = \frac{(1+i)^N - 1}{i(1+i)^N} \)
- \( A = F \cdot (A/F,i,N) = \frac{i}{(1+i)^N - 1} \)
A = P(A/P,i,N) \text{ where } (A/P,i,N) = \frac{i(1+i)^N}{(1+i)^N - 1}

**Wind Energy Calculations**

**Effect of Elevation on Wind Speed**

\[
\tilde{v}_2 = \tilde{v}_1 \left( \frac{h_2}{h_1} \right)^n
\]

where \( n \) is the ground surface friction coefficient and takes on different values according to the nature of the terrain. Some example values are:

- water or smooth flat ground: \( n = 0.1 \)
- tall crops: \( n = 0.2 \)
- city downtown: \( n = 0.4 \)

**Wind Power**

The power of the wind can be calculated from

\[
\dot{W} = \frac{1}{2} \rho A \tilde{v}^3
\]

where

- \( \rho \): density of the air
- \( A \): capture area of the wind
- \( \tilde{v} \): wind speed

**Wind Turbine Efficiency**

The efficiency of a wind turbine is defined as

\[
\eta_{wt} = \frac{\text{wind turbine power produced}}{\text{wind power}} = \frac{\dot{W}_{wt}}{(0.5)\rho A \tilde{v}^3}
\]

Then the power of a wind turbine is given by

\[
\dot{W}_{wt} = \eta_{wt} \frac{1}{2} \rho A \tilde{v}^3
\]

The efficiency for several different types of wind turbines is given in the equations below.

**American Multiblade Wind Turbine**

for \( \text{TSR} \leq 1.75 \): \( \eta_{wt} = 0.39105(\text{TSR})^2 + 0.66586(\text{TSR}) + 0.026583 \)

for \( \text{TSR} > 1.75 \): \( \eta_{wt} = 0 \)

**Darrieus Wind Turbine**

for \( \text{TSR} < 4.6 \): \( \eta_{wt} = 0 \)

for \( 4.6 \leq \text{TSR} \leq 6.86 \): \( \eta_{wt} = -0.078369(\text{TSR})^2 + 0.92146(\text{TSR}) - 2.3532 \)

for \( \text{TSR} > 6.86 \): \( \eta_{wt} = 0 \)
Modern Three-blade Wind Turbine

\[ \text{for } \text{TSR} < 2.95 : \eta_{\text{wt}} = 0 \]
\[ \text{for } 2.95 \leq \text{TSR} \leq 5.4 : \eta_{\text{wt}} = -0.020554(\text{TSR})^2 + 0.18327(\text{TSR}) + 0.023286 \]
\[ \text{for } \text{TSR} > 5.4 : \eta_{\text{wt}} = 0 \]

Ideal Wind Turbine

\[ \text{for } \text{TSR} < 0.5 : \eta_{\text{wt}} = 0.658(\text{TSR}) + 0.023833 \]
\[ \text{for } 0.5 \leq \text{TSR} < 1.0 : \eta_{\text{wt}} = 0.196(\text{TSR}) + 0.23233 \]
\[ \text{for } 1.0 \leq \text{TSR} < 1.5 : \eta_{\text{wt}} = 0.104(\text{TSR}) + 0.32433 \]
\[ \text{for } 1.5 \leq \text{TSR} < 2.5 : \eta_{\text{wt}} = 0.055(\text{TSR}) + 0.399 \]
\[ \text{for } 2.5 \leq \text{TSR} < 4.0 : \eta_{\text{wt}} = 0.022(\text{TSR}) + 0.481 \]
\[ \text{for } \text{TSR} \geq 4.0 : \eta_{\text{wt}} = 0.0041(\text{TSR}) + 0.5532 \]

The tip speed ratio (TSR) is the ratio of the speed at the tip of the wind turbine blade to the wind speed and is given by

\[ \text{TSR} = \frac{\omega R_{\text{rotor}}}{\bar{v}} \]

where

- \( \omega \): rotational speed of the turbine rotor
- \( R_{\text{rotor}} \): radius of the rotor
- \( \bar{v} \): wind speed

Wind Turbine Rotational Speed

To determine the efficiency of a wind turbine from the figure or equations provided above, and iterative approach must be employed as the model employs a set of coupled equations that cannot be solved analytically. The equations of interest for a wind turbine are:

\[ \dot{W} = \eta_{\text{WT}} \frac{1}{2} \rho A \bar{v}^3 \]
\[ \eta_{\text{WT}} = f_n(\text{TSR}) \]
\[ \text{TSR} = \frac{\omega R_{\text{rotor}}}{\bar{v}} \]
\[ \dot{W} = \frac{1}{2} I_{\text{shaft}} \omega^3 \]

where

- \( \dot{W} \): wind turbine power
- \( \eta_{\text{WT}} \): wind turbine efficiency
- \( \rho \): air density
- \( A \): rotor area
- \( \bar{v} \): wind velocity
- \( \text{TSR} \): Tip Speed Ratio
- \( \omega \): wind turbine rotational speed (in radians/sec)
- \( R_{\text{rotor}} \): Radius of the wind turbine rotor
- \( I_{\text{shaft}} \): Moment of inertia of the rotor about its rotating shaft
For our different turbines we have

\[
\text{HAWT: } I_{\text{shaft}} = \frac{N_B \rho_B (L_B W_B t_B) L_B^2}{3}
\]

\[
\text{VAWT: } I_{\text{shaft}} = N_B \rho_B (L_B W_B t_B) R_{\text{rotor}}^2 + \frac{N_B \rho_B (L_B W_B t_B) (W_B^2 + t_B^2)}{12}
\]

where

- \(N_B\): number of blades
- \(\rho_B\): density of blade material
- \(L_B\): length of blade
- \(W_B\): width of blade
- \(t_B\): thickness of blade

Wind Turbine Utilization Factor

The utilization factor (\(U_f\)) or capacity factor for a wind turbine is defined as:

\[
U_f = \frac{\text{Actual Energy Output over Time Period}}{\text{Rated Power x Time Period}}
\]