Section 4.6

4.52 Calculate the equivalent force system for the load given.

Solution: Divide the beam into two loading areas one from 0 to 3 and one from 3 to 7. Using the tables

\[ A_1 = \frac{1}{2}(3)(400) = 600N \quad A_2 = (400)(7 - 3) = 1600N \]

\[ x_1 = \frac{2}{3}(3) = 2m \quad x_3 = 3 + \frac{7 - 3}{2} = 5m \]

The equivalent load of 2200 N acts at the point

\[ x_e = \frac{(2)(600) + 5(1600)}{600 + 1600} = 4.182m \]
4.54 A load of sand is distributed along a truck bed. Compute the equivalent load and its point of application.

![Figure P4.54](image)

Solution: Let $A_1$ denote the area from 0 to 5 ft, $A_2$ denote the rectangle from 5 to 9 ft of height 0.1 ton/ft and let $A_3$ denote the remaining triangle. Then

$$A_1 = (5)(0.2), \quad A_2 = (4)(0.1), \quad A_3 = \frac{1}{2}(4)(0.1)$$

$$x_1 = 5/2, \quad x_2 = 5 + 4/2, \quad x_3 = 5 + 4/3, \quad \sum A_i = 1.6 \text{ tons at}$$

$$x_c = \frac{\sum_i x_i A_i}{\sum_i A_i} = 4.10 \text{ ft}$$
9.2 Determine the second moment of the area $I_{xx}$ about the coordinate axis in figure P9.1.

Solution:

$$I_{xx} = \int dI_{xx} = \int_0^{150} \frac{1}{3} (dx) y^3 = \frac{1}{3} \int_0^{150} y^3 dx = \frac{1}{3} \int_0^{150} \left(80 + \frac{4x}{15}\right)^3 dx$$

$$\frac{1}{3} \left(\frac{80+\frac{4x}{15}}{4}\right)^4 \bigg|_0^{150} = \frac{15}{3(16)} \left(80 + \frac{4x}{15}\right)^4 \bigg|_0^{150} = \frac{15}{3(16)} [120^4 - 80^4]$$

$I_{xx} = 52 \times 10^6 \text{ mm}^4$
9.11 Determine the second moments of the area and the polar moment of inertia about point 0 in figure P9.11.

Solution:

\[ I_{xx} = \int_A (y_0 + 10)^2 \, dA = \int_A y_0^2 \, dA + 2(10) \int_A y_0 \, dA + 10^2 \int_A \, dA = \int_A y_0^2 \, dA + 10^2 \int_A \, dA \]

Since the first moment of inertia about \( CA = 0 \)

\[ I_{xx} = \int_{-\infty}^{\infty} \int_{-10}^{10} y^2 \, dy \, dx + 10^2(20)(120) = \frac{1}{12}(120)(20)^3 + 10^2(120)(20) \]

\[ = 120(20) \left[ \frac{100}{12} + 100 \right] = 10(20)(1600) = 320 \times 10^3 \text{ mm}^4 \]

\[ I_{yy} = \int_A (x_0 + 20)^2 \, dA = \int_A x_0^2 \, dA + 20^2 \int_A \, dA \]

\[ = \frac{1}{12}(20)(120)^3 + 20^2(120)(20) = (120)(20) \left[ \frac{(120)^2}{12} + 20^2 \right] = (120)(20)(1600) = 3840 \times 10^3 \text{ mm}^4 \]

Therefore

\[ I_0 = I_{xx} + I_{yy} = (320 + 3840) \times 10^3 \text{ mm}^4 \]

\[ I_0 = 4160 \times 10^3 \text{ mm}^4 \]
9.32 Determine the second moments of the triangle about a pair of axes parallel to the base and height with origin at the top corner of the triangle in figure P9.31.

Solution:

From P9.31 and using parallel axis theorem

\[ I_{xx_1} = I_{xx_c} + Ad_y^2 = \frac{bh^3}{36} + \left( \frac{bh}{2} \right) \left( \frac{2h}{3} \right)^2 = \frac{bh^3}{36} + \frac{2bh^3}{9} = bh^3 \left( \frac{1}{36} + \frac{2}{9} \right) \]

\[ = \frac{bh^3}{36} (1 + 8) = \frac{3}{4} bh^3 = \frac{bh^3}{4} \]

\[ I_{yy_1} = I_{yy_c} + Ad_x^2 = \frac{1}{48} b^3 h + \left( \frac{bh}{2} \right) \left( \frac{h}{3} \right)^2 = \frac{1}{48} b^3 h + \frac{bh^3}{18} \]

\[ I_{yy_1} = \frac{11bh^3}{144} \]
4.13 Calculate the centroid of the circular arc of radius $R$ centered on the $x$ axis.

Solution: Circular elements are best done in polar coordinates. Following Sample 4.1. Here; $dA = r\,d\theta\,dr$, $x = r\cos\theta$ and $y = r\cos\theta$ so that

$$x_c = \frac{\int x\,dA}{\int dA} = \frac{\int_{-\alpha}^{\alpha} \int_0^R r^2 \cos \theta \,d\theta \,dr}{\int_{-\alpha}^{\alpha} \int_0^R r \,d\theta \,dr} = \frac{2R}{3\alpha} \sin \alpha$$

$$y_c = \frac{\int y\,dA}{\int dA} = \frac{\int_{-\alpha}^{\alpha} \int_0^R r^2 \sin \theta \,d\theta \,dr}{\int_{-\alpha}^{\alpha} \int_0^R r \,d\theta \,dr} = 0$$

Here these integrals are evaluated symbolically.
4.51 The jetway leading to an aircraft is a telescopic device formed by a series of interconnected rectangular parallelepipeds as illustrated. The designer of this device needs to know where the center of gravity is at each position as the jetway telescopes out along the $x$ axis. The length of each rectangular parallelepiped is 20 ft and the walls are 3 inches thick. The first section is $20 \times 10 \times 10$ ft. Using the centerline coordinate system illustrated compute the position of the centroid as the section extends from zero to 15 ft past the edge of the first section.

**FIGURE P4.51**

Solution: The $y$ and $z$ axis are on center lines which are each also an axis of symmetry (i.e., $y_c = z_c = 0$, in the coordinate system given) so it remains to compute $x_c$. The first box has volume

$$V_1 = (20 \times 10 \times 10) - (20 - .5)(10 - .5)(20) = 195 \text{ ft}^3$$

with $x$ centered, fixed at

$$x_1 = \frac{20}{2} = 10 \text{ ft}.$$

The volume of the second section is

$$V_2 = (20 - .5)(10 - .5)(20) - (9)(9)(20) = 185 \text{ ft}^3.$$

Now the $x$ centroid of section 2 depends on how far its extended, i.e., $x_2 = 10 + \ell$ where $\ell$ is the distance the second section extends out of (continued)
the first. Using the composite volume formula calculate

\[ x_c = \frac{\sum_{i=1}^{2} x_i V_i}{V_1 + V_2} = \frac{(10)(195) + (10 + \ell)(185)}{185 + 185} \]

so

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describes the center of mass as section 2 telescopes out from section 1.
4.60 A side impact air bag for an automobile may be modeled as a
distributed load by \( w(x) = 100(1 + x \cos x) \) lb/ft. Compute the
equivalent force and point of action for this system.

![Diagram](image)

(a) schematic  (b) sketch  (c) load model

FIGURE P4.60

Solution:

\[
W = \int_0^2 100(1 + x \cos x)dx = 240.2 \text{ lb}
\]

\[
x_c = \frac{1}{W} \int_0^2 100x (1 + x \cos x)dx = .897 \text{ ft}
\]

(Solution computed via mathcad.)