4.13 Calculate the centroid of the circular arc of radius $R$ centered on the $x$ axis.

Solution: Circular elements are best done in polar coordinates. Following Sample 4.1. Here, $dA = r\,d\theta\,dr$, $x = r \cos \theta$ and $y = r \cos \theta$ so that

$$
x_c = \frac{\int x\,dA}{\int dA} = \frac{\int_{-\alpha}^{\alpha} \int_0^R r^2 \cos \theta \,d\theta\,dr}{\int_{-\alpha}^{\alpha} \int_0^R r \,d\theta\,dr} = \frac{2R}{3\alpha} \sin \alpha
$$

$$
y_c = \frac{\int y\,dA}{\int dA} = \frac{\int_{-\alpha}^{\alpha} \int_0^R r^2 \sin \theta \,d\theta\,dr}{\int_{-\alpha}^{\alpha} \int_0^R r \,d\theta\,dr} = 0
$$

Here these integrals are evaluated symbolically.