2.61 Calculate the vector $\mathbf{C}$, where $\mathbf{C} = \mathbf{A} + \mathbf{B}$ and $\mathbf{A}$ and $\mathbf{B}$ are given as $\mathbf{A} = \mathbf{i} + 2\mathbf{k}$ and $\mathbf{B} = -2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$. Form a unit vector in the direction of $\mathbf{C}$ and write $\mathbf{C}$ in terms of this unit vector.

![Diagram of vectors A, B, and C]

Solution:

$$\mathbf{C} = \mathbf{A} + \mathbf{B} = -\mathbf{i} + 3\mathbf{j} + 3\mathbf{k},$$

so that

$$|\mathbf{C}| = \sqrt{1^2 + 3^2 + 3^2} = \sqrt{19} = 4.35$$

and

$$\lambda_x = \cos \theta_x = \frac{-1}{4.35} = -0.229,$$

$$\lambda_y = \cos \theta_y = \frac{3}{4.35} = 0.6896 = 0.690 = \lambda_x.$$ 

Thus

$$\hat{e}_c = -0.229\mathbf{i} + 0.690\mathbf{j} + 0.690\mathbf{k}$$

and

$$\mathbf{C} = 4.35\hat{e}_c.$$ 

Using Mathcad as a vector calculator by copying 2.60, pasting and changing the entries of $\mathbf{A}$ and $\mathbf{B}$ yields:

$$A := \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \quad B := \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \quad C := A + B \quad ec := \frac{C}{|C|} \quad C = \begin{pmatrix} -1 \\ 3 \\ 3 \end{pmatrix}$$

$$ec = \begin{pmatrix} -0.229 \\ 0.69 \\ 0.69 \end{pmatrix} \quad |C| \cdot ec = \begin{pmatrix} -1 \\ 3 \\ 3 \end{pmatrix} |C| = 4.35 \quad \blacksquare$$

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2.64 A net force of \( R = 1000 \) N down the pole results from the guy wires sketched in the figure. If the pole is 10 m high calculate the components of \( R \) along each of the wires.

\[
\begin{align*}
A = -5\hat{i} + 10\hat{j} - 5\hat{k}
\end{align*}
\]

so that a unit vector along \( A \) is

\[
\hat{e}_A = \frac{A}{|A|} = \frac{1}{\sqrt{5^2 + 10^2 + 5^2}} A = -0.4082\hat{i} + 0.8165\hat{j} - 0.4082\hat{k}.
\]

Likewise

\[
\hat{e}_B = \frac{1}{\sqrt{5^2 + 10^2}} (5\hat{i} + 10\hat{j}) = 0.4472\hat{i} + 0.8944\hat{j}
\]

and

\[
\hat{e}_c = \frac{1}{\sqrt{10^2 + 5^2}} (10\hat{j} + 5\hat{k}) = 0.8944\hat{j} + 0.4472\hat{k}.
\]

The vector \( \mathbf{R} \) can now be written as the sum of components along these directions i.e.,

\[
\mathbf{R} = R_A\hat{e}_A + R_B\hat{e}_B + R_c\hat{e}_c
\]

where \( R_A, R_B \) and \( R_c \) are the unknown but desired component values. Writting out this last expression where \( R = 1000\hat{j} \) N yields

\[
0\hat{i} + 1000\hat{j} + 0\hat{k} = R_A(-0.4082\hat{i} + 0.8165\hat{j} - 0.4082\hat{k}) + R_B(0.4472\hat{i} + 0.8944\hat{j}) + R_c(0.8944\hat{j} + 0.4472\hat{k}).
\]

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Comparing coefficients of each unit vector yields

\[ 0 = -0.4082R_A + 0.4472R_B + 0R_c, \]

from \( \hat{i} \),

\[ 1000 = .8165R_A + .8944R_B + .8944R_c, \]

from \( \hat{j} \),

\[ 0 = -0.4082R_A + 0R_B + .4472R_c, \]

from \( \hat{k} \) which can be written in matrix form as

\[
\begin{bmatrix}
0 \\
1000 \\
0
\end{bmatrix}
= \begin{bmatrix}
-0.4082 & 0.4472 & 0 \\
0.8165 & 0.8944 & 0.8944 \\
-0.4082 & 0 & 0.4472
\end{bmatrix}
\begin{bmatrix}
R_A \\
R_B \\
R_c
\end{bmatrix}
= ar
\]

Solving by typing \( r = inv(a) \times b \) in MATLAB results in

\[
\begin{bmatrix}
408.2799 \\
372.6741 \\
372.6791
\end{bmatrix}
\text{ or } R_A = 408.3, \ R_B = R_c = 372.7 \text{ N}
\]

and the vector \( \mathbf{R} \) may be written as

\[
\mathbf{R} = 408.3 \mathbf{e}_A + 372.7 \mathbf{e}_B + 372.7 \mathbf{e}_c
\]

In Mathcad this is solved as follows:

\[
A := \begin{bmatrix}
-5 \\
10 \\
-5
\end{bmatrix}
\]

\[
e_a := \frac{A}{|A|}
\]

\[
C := \begin{bmatrix}
5 \\
10 \\
0
\end{bmatrix}
\]

\[
e_b := \frac{Bb}{|B|}
\]

\[
C := \begin{bmatrix}
0 \\
10 \\
5
\end{bmatrix}
\]

\[
e_c := \frac{C}{|C|}
\]

\[
T1 := \text{augment} \ (e_a, e_b)
\]

\[
T := \text{augment} \ (T1, e_d)
\]

\[
E := T^{-1} \cdot \begin{bmatrix}
0 \\
1000 \\
0
\end{bmatrix}
\]

\[
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\]
\[
R = \begin{pmatrix}
408.248 \\
372.678 \\
372.678
\end{pmatrix}
\]

which agrees with the above. Note for comparison:

\[
T = \begin{pmatrix}
-0.408 & 0.447 & 0 \\
0.816 & 0.894 & 0.894 \\
-0.408 & 0 & 0.447
\end{pmatrix}
\]

\[
\]
2.70 Solve the following system of equations:

\[
\begin{align*}
T_1 - T_2 + 3T_3 - 2T_4 &= 100 \\
6T_2 - 2T_3 + T_4 - T_5 &= -60 \\
-2T_1 + 5T_2 + T_3 + 6T_5 &= 200 \\
-T_1 - 4T_3 - 3T_4 + T_5 &= -40 \\
-T_1 - 3T_2 + 2T_3 + T_4 - 3T_5 &= 150
\end{align*}
\]

Solution: In MATLAB enter the coefficient matrix

\[
A = \begin{bmatrix}
1 & -1 & 3 & -2 & 0 \\
0 & 6 & -2 & 1 & -1 \\
-2 & 5 & 1 & 0 & 6 \\
-1 & 0 & -4 & -3 & 1 \\
-1 & -3 & 2 & 1 & -3
\end{bmatrix}
\]

\[
b = \begin{bmatrix}
100 \\
-60 \\
200 \\
-40 \\
150
\end{bmatrix}
\]

then type \(A\backslash b\) to get

\[
T = \begin{bmatrix}
-81.0604 \\
8.2737 \\
46.6708 \\
-24.6609 \\
-8.3600
\end{bmatrix}
\]

where the top element is \(T_1\), the second \(T_2\), etc.
2.71 For the matrices \( [A] = \begin{bmatrix} 4 & -2 & 0 \\ -1 & 3 & 2 \\ 1 & 0 & 2 \end{bmatrix} \) and \( [B] = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 2 & -2 \\ -3 & -1 & 2 \end{bmatrix} \), determine a) \( [A] + [B] \), b) \( [B] - [A] \).

Solution: (solved in MATLAB)

a) \( A + B = \begin{bmatrix} 5 & 1 & 1 \\ -1 & 5 & 0 \\ -2 & -1 & 4 \end{bmatrix} \)

b) \( B - A = \begin{bmatrix} -3 & 5 & 1 \\ 1 & -1 & -4 \\ -4 & -1 & 0 \end{bmatrix} \)

2.72 For the matrices in Problem 2.71, determine a) \( [A][B] \), b) \( [B][A] \).

Solution: (solved in MATLAB)

\[ AB = \begin{bmatrix} 4 & 8 & 8 \\ -7 & 1 & -3 \\ -5 & 1 & 5 \end{bmatrix}, \quad BA = \begin{bmatrix} 2 & 7 & 8 \\ -4 & 6 & 0 \\ -9 & 3 & 2 \end{bmatrix} \]

2.73 Find the determinant of \( [A] \) and \( [B] \) in Problem 2.71.

Solution: (solved in MATLAB)

\[ \det A = 16, \quad \det B = 26 \]
2.82 Consider the vector \( \mathbf{R} = -\hat{i} + 3\hat{j} \) and compute the component of \( \mathbf{R} \) in the direction of the line \( V \) and a second component perpendicular to the line \( V \).

![Figure P2.82](image)

Solution: \( \hat{e}_v \) up along \( v \) is

\[
\hat{e}_v = \cos 60\degree \hat{i} + \sin 60\degree \hat{j} = 0.5\hat{i} + 0.8660\hat{j}.
\]

Let \( R_v \) denote the projection of \( F \) onto \( \hat{e}_v \), i.e.,

\[
R_v = \mathbf{R} \cdot \hat{e}_v = (.5)(-1) + (.866)(.3) = +2.098.
\]

So that a vector component of \( \mathbf{R} \) along \( v \), denoted \( \mathbf{R}_v \) is

\[
\mathbf{R}_v = (2.098)\hat{e}_v = 2.098(.5\hat{i} + .866\hat{j}) = +1.049\hat{i} + 1.817\hat{j}.
\]

Now \( \mathbf{R} \) can be written as two components, one parallel to \( v \) as \( \mathbf{R}_v \) and one perpendicular denoted \( \mathbf{R}_\perp \). These 3 vectors must satisfy

\[
\mathbf{R} = \mathbf{R}_v + \mathbf{R}_\perp
\]

or

\[
\mathbf{R}_\perp = \mathbf{R} - \mathbf{R}_v = (-\hat{i} + 3\hat{j}) - (1.049\hat{i} + 1.817\hat{j}).
\]

Thus

\[
\mathbf{R}_\perp = -2.049\hat{i} + 1.183\hat{j}.
\]

To check note that

\[
\mathbf{R}_\perp \cdot \mathbf{R}_v = 0.
\]

A solution via Mathcad proceeds as follows:

\[
ev := \begin{pmatrix} \cos(60 \cdot \text{deg}) \\ \sin(60 \cdot \text{deg}) \\ 0 \end{pmatrix} \quad r := \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix} \quad R_v := (\mathbf{R} \cdot \mathbf{ev}) \cdot \mathbf{ev} \quad Rp := \mathbf{R} - R_v
\]
\[ R_v = \begin{pmatrix} -1.049 \\ 1.817 \\ 0 \end{pmatrix} \quad R_p = \begin{pmatrix} -2.049 \\ 1.183 \\ 0 \end{pmatrix} \quad \text{Note that } R_p \cdot R_v = 0 \]
2.86 Resolve the vector \( \mathbf{F} \) into the sum of a component along \( \mathbf{a} \) and one along \( \mathbf{b} \); and compare these components to the projection of \( \mathbf{F} \) onto \( \mathbf{a} \) and the projection of \( \mathbf{F} \) onto \( \mathbf{b} \).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure.png}
\caption{FIGURE P2.86}
\end{figure}

Solution: To resolve \( \mathbf{F} \) into nonorthogonal components along \( \mathbf{a} \) and \( \mathbf{b} \) the dot product cannot be used. Rather solve for the unit vector \( \hat{\mathbf{a}} \) and \( \hat{\mathbf{b}} \) along \( \mathbf{a} \) and \( \mathbf{b} \) and find two scalars \( F_a \) and \( F_b \) such that

\[ \mathbf{F} = F_a \hat{\mathbf{a}} + F_b \hat{\mathbf{b}} \]

\[ \hat{\mathbf{a}} = \frac{1}{\sqrt{4^2 + 5^2}} (4\mathbf{i} + 5\mathbf{j}) = .6247\mathbf{i} + .7809\mathbf{j} \]

and

\[ \hat{\mathbf{b}} = \frac{1}{\sqrt{8^2 + 2^2}} (8\mathbf{i} + 2\mathbf{j}) = .9701\mathbf{i} + .2425\mathbf{j} . \]

Solve

\[ \mathbf{F} = F_a \hat{\mathbf{a}} + F_b \hat{\mathbf{b}} \]

yields

\[ 5\mathbf{i} + 4\mathbf{j} = (.6247F_a + .9701F_b)\mathbf{i} + (.7809F_a + .2425F_b)\mathbf{j} \]

or upon equating coefficients:

\[
\begin{bmatrix}
5 \\
4
\end{bmatrix} = \begin{bmatrix}
.6247 & .9701 \\
.7809 & .2425
\end{bmatrix} \begin{bmatrix}
F_a \\
F_b
\end{bmatrix} .
\]

This yields \( F_a = 4.402 \), \( F_b = 2.319 \) (via MATLAB). Thus

\[ \mathbf{F} = 4.402\hat{\mathbf{a}} + 2.319\hat{\mathbf{b}} \]

where

\[ \hat{\mathbf{a}} = .6247\mathbf{i} + .7809\mathbf{j} \]

and

\[ \hat{\mathbf{b}} = .9701\mathbf{i} + .2425\mathbf{j} . \]

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Next consider the projection of \( \mathbf{F} \) along \( \mathbf{b} \) determined by

\[
\mathbf{F} \cdot \hat{e}_b = (5\mathbf{i} + 4\mathbf{j}) \cdot (.9701\mathbf{i} + .2425\mathbf{j}) = 5.82
\]

which of course is not equal to \( F_b = 2.319 \). Next consider the projection of \( \mathbf{F} \) along \( \mathbf{a} \) determined by

\[
F_a = \mathbf{F} \cdot \hat{e}_a = (5\mathbf{i} + 4\mathbf{j}) \cdot (.6247\mathbf{i} + .7809\mathbf{j}) = 6.25
\]

which of course is not equal to the component magnitude \( F_a = 4.402 \). Using Mathcad the solution becomes:

\[
F := \begin{pmatrix} 5 \\ 4 \end{pmatrix} \quad a := \begin{pmatrix} 4 \\ 5 \end{pmatrix} \quad b := \begin{pmatrix} 8 \\ 2 \end{pmatrix}
\]

\[
ea := \frac{a}{|a|} \quad eb := \frac{b}{|b|} \quad C := \text{augment} \ (ea, \ eb)
\]

\[
\begin{pmatrix} F_a \\ F_b \end{pmatrix} := C^{-1} \cdot F \quad Fa = 4.402 \quad Fb = 2.319
\]

\[
ea = \begin{pmatrix} 0.625 \\ 0.781 \end{pmatrix} \quad eb = \begin{pmatrix} 0.97 \\ 0.243 \end{pmatrix}
\]

\[\blacksquare\]
2.94 The positions of the markers on the gymnast’s leg while landing after an aeral maneuver are measured relative to a coordinate system in the laboratory. As known from watching the olympics, the angle the knee makes upon landing is one of the factors used by judges. In training athletes, one often use markers to measure these angles by measuring the locations relative to the laboratory frame of reference. Some sample measurements are:

<table>
<thead>
<tr>
<th></th>
<th>x (meters)</th>
<th>y (meters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hip</td>
<td>2.10</td>
<td>1.0</td>
</tr>
<tr>
<td>Knee</td>
<td>2.30</td>
<td>0.6</td>
</tr>
<tr>
<td>Ankle</td>
<td>2.15</td>
<td>0.15</td>
</tr>
</tbody>
</table>

**FIGURE P2.94**

Determine a) the length of the gymnast’s leg; b) the angle of flexion of the knee, that is the angle between the lower leg and thigh.

Solution: a) Using the data given, the vector from the hip, \( \mathbf{r}_h \) is

\[
\mathbf{r}_h = 2.1\hat{i} + \hat{j}(m),
\]

that to the knee is

\[
\mathbf{r}_k = 2.3\hat{i} + .6\hat{j}(m)
\]

and that to the ankle is

\[
\mathbf{r}_a = 2.15\hat{i} + .15\hat{j}.
\]

Thus the distance from the hip to the knee is the vector

\[
\mathbf{r}_a = \mathbf{r}_h - \mathbf{r}_k = (2.1 - 2.3)\hat{i} + (1 - .6)\hat{j} = -.2\hat{i} + .4\hat{j}
\]

which has magnitude

\[
\sqrt{(-.2)^2 + (.4)^2} = .447 \text{ m.}
\]
Likewise the vector describing the lower leg is

$$\mathbf{r}_l = \mathbf{r}_k - \mathbf{r}_a = (2.3 - 2.15)\hat{i} + (.6 - .15)\hat{j} = .15\hat{i} + .45\hat{j}$$

and

$$|\mathbf{r}_l| = \sqrt{.15^2 + .45^2} = .474 \text{ m.}$$

The total length of the dancer’s leg is the distance between the hip and the ankle when the angle between them is zero so

$$\ell = .447 + .474 = 0.922 \text{ m.}$$

b) At the configuration illustrated the angle of flex = $\theta$ where

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_a \cdot \mathbf{r}_l}{|\mathbf{r}_a||\mathbf{r}_l|}\right) = \left[\frac{-.2(.15) + (.4)(.45)}{.474(.447)}\right] = 45^\circ.$$ 

Using Mathcad the solution becomes

$$rh := \begin{pmatrix} 2.1 \\ 1 \end{pmatrix} \quad rk := \begin{pmatrix} 2.3 \\ .6 \end{pmatrix} \quad ra := \begin{pmatrix} 2.15 \\ .15 \end{pmatrix}$$

$$ru := rh - rk \quad ru = \begin{pmatrix} -.2 \\ 0.4 \end{pmatrix} \quad |ru| = 0.44$$

$$rl := rk - ra \quad rl = \begin{pmatrix} 0.15 \\ 0.45 \end{pmatrix} \quad |rl| = 0.474 \quad 1 := |rl| + |ru| \quad 1 = 0.922$$

$$\theta := \frac{acos\left(\frac{ru \cdot rl}{|ru||rl|}\right)}{deg}, \quad \theta = 45, \quad ru \cdot rl = 0.15$$
PROBLEMS FOR SECTION 2.10

2.105 A block of mass \( m = 1000 \) kg is suspended from a three-cable arrangement as illustrated. For the case that \( \alpha = 30^\circ \) and \( \beta = 45^\circ \), compute the tensions \( T_1 \), \( T_2 \) and \( T_3 \).

![Diagram of a block suspended by three cables](image)

FIGURE P2.105

Solution: Two free body diagrams are required. A free body diagram of the mass \( m \) yields S2.94a and of the point \( A \) yields figure S2.94b.

From S2.94a the equilibrium equation in the \( y \) direction yields \( T_1 - mg = 0 \) or

\[
T_3 = (1000)(9.87) = 9810 \text{ N}, \quad \text{or} \quad \mathbf{T}_3 = 9810\hat{j} \text{ N},
\]

the equilibrium equations for S2.94b are

\[
\mathbf{T}_1 + \mathbf{T}_2 + \mathbf{T}_3 = 0
\]

In vector component form this becomes

\[
-9810\hat{j} + T_2 \cos 45^\circ \hat{i} + T_2 \sin 45^\circ \hat{j} - T_1 \cos 30^\circ \hat{i} + T_1 \sin 30^\circ \hat{j} = 0
\]

Upon writing this as two scalar equations by equating coefficients of \( \hat{i} \) then \( \hat{j} \) this becomes

\[
\begin{bmatrix}
\cos 45^\circ & -\cos 30^\circ \\
\sin 45^\circ & \sin 30^\circ
\end{bmatrix}
\begin{bmatrix}
T_2 \\
T_1
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
9810
\end{bmatrix}
\]

which can be solved by a simple matrix inversion (expressed in Newtons) from Mathcad.

\[
\begin{pmatrix}
T_2 \\
T_1
\end{pmatrix}
:=
\begin{pmatrix}
\cos(45 \cdot \text{deg}) & -\cos(30 \cdot \text{deg}) \\
\sin(45 \cdot \text{deg}) & \sin(39 \cdot \text{deg})
\end{pmatrix}^{-1} \cdot 
\begin{pmatrix}
0 \\
9810
\end{pmatrix}
= 
\begin{pmatrix}
8.795 \times 10^3 \\
7.181 \times 10^3
\end{pmatrix} \text{ N}
\]

\[
T_2 = 8.795 \times 10^3, \quad T_1 = 7.181 \times 10^3.
\]

\[
\blacksquare
\]
2.113 A typical load on a traffic light, resulting from both gravity and wind, is modeled by $\mathbf{F} = -25\mathbf{i} - 500\mathbf{j} + 25\mathbf{k}$ (lb) acting at the top of the light. Determine the tension in the support cables. The dimensions are given in feet.

![Figure P2.113](image)

**FIGURE P2.113**

Solution: A free body diagram of the connection point is given in figure S2.102. The equilibrium equation is

$$\mathbf{T}_1 + \mathbf{T}_2 + \mathbf{T}_3 + \mathbf{F} = 0$$

![Figure S2.113](image)

**FIGURE S2.113**

Each $T_i$ can be written as a magnitude indicated by the geometry. Then the solution is given by the matrix inversion

$$
\begin{pmatrix}
T_1 \\
T_2 \\
T_3
\end{pmatrix} = - \left( \hat{e}_1; \hat{e}_2; \hat{e}_3 \right)^{-1} \left( \hat{e}_1; \hat{e}_2; \hat{e}_3 \right)^{-1} F = \left( \hat{e}_1; \hat{e}_2; \hat{e}_3 \right)^{-1} \begin{pmatrix} 25 \\ 500 \\ -25 \end{pmatrix}
$$

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\[ \dot{e}_1 = \frac{r_1}{|r_1|} = \frac{1}{\sqrt{10^2 + 4^2 + 2^2}} \left( \frac{-10}{4} \right) \]

\[
\begin{align*}
    r_1 &:= \begin{pmatrix} -10 \\ 4 \\ 2 \end{pmatrix} \\
    e_1 &:= \frac{r_1}{|r_1|} \\
    r_2 &:= \begin{pmatrix} 0 \\ 4 \\ -1 \end{pmatrix} \\
    e_2 &:= \frac{r_2}{|r_2|} \\
    r_3 &:= \begin{pmatrix} 10 \\ 4 \\ 1 \end{pmatrix} \\
    e_3 &:= \frac{r_3}{|r_3|} \\
    m &:= \text{augment} (e_1, e_2) \\
    c &:= \text{augment} (m, e_3) \\
    \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix} &:= c^{-1} \begin{pmatrix} 25 \\ 500 \\ -25 \end{pmatrix} \quad T_1 = 208.135, \ T_2 = 348.402, \ T_3 = 232.558 \text{ lb}
\end{align*}
\]