Problem 1. (10 points)
A 2000 kg automobile starts from rest at point A on a 6 deg incline and coasts (without any friction) through a distance of 150 m to point B. The brakes are then applied, causing the automobile to come to a stop at point C, 20 m from B. Neglecting air resistance, find:
(a) the speed of the automobile at point B,
(b) coefficient of friction between the tires and the road.

Problem 2. (10 points)
A train made of two cars travels at 72 km/hr. The mass of car A is 18 Mg and the mass of car B is 13 Mg. When brakes are suddenly applied, a constant braking force of 19 kN is applied to each car. Determine (a) the time required for the train to stop after the brakes are applied, (b) the force in the coupling between the cars while the train is slowing down.

Problem 3. (10 points)
Knowing that the disk has a constant angular velocity of 15 rad/sec clockwise, determine the angular velocity of bar BD and velocity of the collar D when
(a) $\theta = 0$ deg, and (b) $\theta = 90$ deg.
Find the instantaneous center of rotation of bar BD when $\theta = 0$ deg.
\( T_A + U_{A \rightarrow B} = T_B \)

\( T_A = 0 \)

\( U_{A \rightarrow B} = mg \sin 6^\circ \times 150 = 2000 \times 9.81 \times \sin 6^\circ \times 150 \)

\[ = 307627.27 \text{ N-m} \]

\( T_B = U_{A \rightarrow B} \)

\[ T_B = \frac{1}{2} m v_B^2 = \frac{1}{2} \times 2000 \times v_B^2 = 1000 v_B^2 \]

\[ \rightarrow 1000 v_B^2 = 307627.27 \rightarrow v_B = 17.54 \text{ m/sec} \]

(b) \( T_A + U_{A \rightarrow C} = T_C \)

\( T_A = 0 \), \( T_C = 0 \) \( \rightarrow U_{A \rightarrow C} = 0 \)

\( U_{A \rightarrow C} = mg \sin 6^\circ \times 170 - F_s \times 20 = 0 \)

\[ \rightarrow F_s = \frac{2000 \times 9.81 \times \sin 6^\circ \times 170}{20} \]

\[ = 17432.212 \text{ N} \]

\( F_s = MN = M \cdot mg \cos 6^\circ = M \cdot 2000 \times 9.81 \times \cos 6^\circ \)

\[ \rightarrow M = 0.893 \]
Look at both cars together,

\[ m v_1 + I_{1-2} = m v_2 \]

where \( m = (18 + 13) \times 10^3 \text{ kg} \)

\[ 31000 \times \frac{72 \times 1000}{3600} + I_{1-2} = 0 \]

\[ I_{1-2} = -(38000) t_{1-2} \]

\[ \rightarrow 31000 \times \frac{72000}{3600} = 38000 t_{1-2} \]

\[ \rightarrow t_{1-2} = 16.31 \text{ sec} \]

Look at car A alone

Let \( T = \) force in coupling.

\[ m v_1 + I_{1-2} = m v_2 \]

\[ 18000 \times \frac{72000}{3600} - (19000 + T) t_{1-2} = 0 \]

or \[ 18000 \times 20 = (19000 + T) \times 16.31 \]

\[ \rightarrow T = 3072.3 \text{ N} \]
#3. \[ \vec{U}_B = \vec{U}_A + \vec{\omega}_{disk} \times \vec{V}_{B/A} \]

\[ = 0 + (-15 \hat{r}) \times 2.8 (\cos \theta \hat{j} + \sin \theta \hat{i}) \]

\[ = 15 \times 2.8 \cos \theta \hat{i} - 15 \times 2.8 \sin \theta \hat{j} \]

\[ = 42 \cos \theta \hat{i} - 42 \sin \theta \hat{j} \]

\[ \vec{U}_D = \vec{U}_B + \vec{\omega}_{BD} \times \vec{T}_{D/B} \]

\[ \vec{U}_D \hat{j} = 42 \cos \theta \hat{i} - 42 \sin \theta \hat{j} + \omega_{BD} \hat{k} \times 10 [- \cos \theta \hat{j} - \sin \theta \hat{i}] \]

\[ \vec{U}_D \hat{j} = 42 \cos \theta \hat{i} - 42 \sin \theta \hat{j} + 10 \omega_{BD} \cos \alpha \hat{i} \]

\[ - 10 \omega_{BD} \sin \alpha \hat{j} \]

where \( \alpha = \text{defined in figure in page 1} \)

\[ \sin \alpha = \frac{2.8 + 2.8 \sin \theta}{10} = 0.28(1 + \sin \theta) \]

From (1):

\[ \vec{U}_D = -42 \sin \theta - 10 \omega_{BD} \sin \alpha \]

\[ \vec{U}_D = -42 \sin \theta - 10 \omega_{BD} \times 0.28 (1 + \sin \theta) \]

and \( \theta = 42 \cos \theta + 10 \omega_{BD} \cos \alpha \)

(a) for \( \theta = 0 \), (2) & (3) gives:

\[ \vec{U}_D = -10 \omega_{BD} \times 0.28 \]

\[ 0 = 42 + 10 \omega_{BD} \cos \alpha \]

where \( \sin \alpha = 0.28 \)

\[ \Rightarrow \omega_{BD} = 4.375 \text{ rad/sec} \]

\[ \vec{U}_D = 12.25 \text{ m/sec} \]
(b) for $\theta = 90^\circ$, (2) and (3) gives:

$$\sin \alpha = 0.28 \times 2 = 0.56 \quad \rightarrow \quad \alpha = 34.05^\circ$$

$$V_D = -42 -10 \omega_{BD} \times 0.28 \times 2$$

$$0 = 10 \omega_{BD} \cos \alpha$$

$$\rightarrow \omega_{BD} = 0 \quad , \quad V_D = 42 \text{ m/sec} \downarrow$$

for $\theta = 0$, the instantaneous center of rotation of link BD is shown below by point C.