ME 201
Thermodynamics

Handout: Conservation of Mass

The general form of our conservation of mass equation is:

$$\frac{dm_{sys}}{dt} = \sum_{inflows} \dot{m}_{in} - \sum_{outflows} \dot{m}_{out}$$

where

$\frac{dm_{sys}}{dt}$: change in mass within the system per time
$\sum_{inflows} \dot{m}_{in}$: sum of all the mass inflows to the system
$\sum_{outflows} \dot{m}_{out}$: sum of all the mass outflows from the system

Occasionally, it will be useful to explode the derivative in our equation to differences, so we may write

$$\frac{m_{final} - m_{final}}{\Delta t} = \sum_{inflows} \dot{m}_{in} - \sum_{outflows} \dot{m}_{out}$$

where

m_{final}: final mass in system after process
$m_{initial}$: initial mass in system before process
Δt: time change during process

Continuity Relations

When a flowing system, such as a river or jet aircraft engine, is considered it is convenient to introduce some flow parameters in addition to the mass flow rate. We define

$$\dot{V} : \text{volume flow rate (e.g., m}^3\text{/s)}$$
$$\rho : \text{fluid density (e.g., kg/m}^3\text{)}$$
$$\vec{v} : \text{fluid velocity (e.g., m/s)}$$
$$A_c : \text{flow cross-sectional area (e.g., m}^2\text{)}$$

There are a number of relationships among these parameters and the mass flow rate, given by:

$$\dot{m} = \rho \cdot \dot{V} = \rho \cdot A_c \cdot \vec{v}$$
$$\dot{V} = \frac{\dot{m}}{\rho} = A_c \cdot \vec{v}$$
$$\vec{v} = \frac{\dot{m}}{\rho \cdot A_c} = \frac{\dot{V}}{A_c}$$

If we consider two locations in a flowing system like a river, our conservation of mass equation can be written

$$\dot{m}_1 = \dot{m}_2$$

or we may write

$$(\rho \cdot A_c \cdot \vec{v})_1 = (\rho \cdot A_c \cdot \vec{v})_2$$

which is called our continuity equation.

<u>Types of Thermodynamic Systems</u>

We define three type of thermodynamics systems as related to terms in the conservation of mass equation.

<u>Closed System: No mass inflows or outflows or</u>

$$\sum_{\text{inflows}} \dot{m}_{in} = \sum_{\text{outflows}} \dot{m}_{out} = 0$$

which gives

$$\frac{dm_{sys}}{dt} = 0$$

<u>Control Volume System: All mass that flows in, flows out or</u>

$$\sum_{\text{inflows}} \dot{m}_{in} = \sum_{\text{outflows}} \dot{m}_{out}$$

which gives

$$\frac{dm_{sys}}{dt} = 0$$

Hence, there is no accumulation or depletion of mass in the system.

<u>Transient System: General conservation of mass equation applies or</u>

$$\frac{dm_{sys}}{dt} = \sum_{\text{inflows}} \dot{m}_{in} - \sum_{\text{outflows}} \dot{m}_{out}$$

Hence, we can have multiple mass inflows and outflows that will lead to accumulation or depletion of mass in the system.