ME 201
Thermodynamics

Solutions to Transient System Practice Problems

1. A balloon initially contains 5 m^3 CO_2 at 100 kPa and 22°C. It is connected to a CO_2 gas line that provides CO_2 at 170 kPa and 30°C. The balloon is then filled to a pressure of 170 kPa. During this process the balloon is insulated and it is observed that the pressure and total volume are related by

   PV^{-1} = constant

   Determine the final temperature and volume of the balloon.

Solution:

We begin by setting up our table.

System Type: Transeint System
Substance Type: Ideal gas
Process: Polytropic with n=-1
State 1: Fixed
State 2: Unknown
State In: Fixed

$$W_{sh} = 0$$

$$Q = 0$$

$$W_{bnd} = \frac{P_2V_2 - P_1V_1}{1-n}$$

Conservation of mass: $m_2 - m_1 = m_{in}$

1st Law: $m_2u_2 - m_1u_1 = m_{in}h_{in} - W_{bnd}$

State 1	**State In**	**State 2**
P_1 = 100 kPa	P_{in} = 170 kPa	P_2 = 170 kPa
T_1 = 295 K	T_{in} = 303 K	T_2 = *324 K*
V_1 = 5 m^3		V_2 = **8.5 m^3**
m_1 = *8.973 kg*	m_{in} = **14.71 kg**	m_2 = **23.6 kg**
u_1 = *154.49 kJ/kg*	h_{in} = *216.87 kJ/kg*	u_2 = ***173.89 kJ/kg***

Italicized values are obtained from N_2 tables or ideal gas equation.
Bold values are calculated.

At state 1 we know both the temperature and pressure, so the state is fixed. We can calculate the mass from

$$m_1 = \frac{P_1V_1}{RT_1} = \frac{(100)(5)}{(0.1889)(295)} = 8.973\,\text{kg}$$

The internal energy can be read from the CO_2 tables, but will have to be converted to kJ/kg or

$$u_1 = \frac{\bar{u}_1}{MW_{N_2}} = \frac{6799}{44.01} = 154.49\ \text{kJ/kg}$$

At the inlet state we can find the enthalpy from the CO_2 tables or

h_{in} = 216.87 kJ/kg

To begin to fix our final state, we use the polytropic relation or

$$P_2V_2^{-1} = P_1V_1^{-1}$$

or

$$V_2 = V_1\frac{P_2}{P_1} = (5)\frac{170}{100} = 8.5\ \text{m}^3$$

The boundary work can now be calculated

$$W_{bnd} = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{(170)(8.5) - (100)(5)}{1 - (-1)}$$

$$= 472.5\ \text{kJ}$$

For the mass at state 2, we have

$$m_2 = \frac{P_2V_2}{RT_2}$$

But since we do not have the temperature we cannot solve for this. The temperature could come from u_2 as calculated from the 1st law, but the 1st law contains m_2, so that we have three equations and three unknowns. We are going to have to solve this by trial error as follows:

Guess a T_2

Calculate m_2, $m_2 = \frac{P_2V_2}{RT_2}$

Calculate m_{in}, $m_{in} = m_2 - m_1$

Calculate u_2, $u_2 = \frac{1}{m_2}\left[m_1u_1 + m_{in}h_{in} - W_{bnd}\right]$

Look T_2 up in the table corresponding to u_2

Repeat loop until T_2 does not change

Now starting

Guess T_2 = 300 K

Calculate m_2, $m_2 = \dfrac{(170)(8.5)}{(0.1889)(300)} = 25.5 \text{ kg}$

Calculate m_{in}, m_{in} = 25.5 - 8.973 = 16.526 kg

Calculate u_2,

$$u_2 = \frac{1}{25.5}\left[(8.973)(154.49) + (16.526)(216.87) - 472.5\right] = 176.38 \text{ kJ/kg}$$

Look T_2 = 327 K

Repeat loop with T_2 = 328 K

Guess T_2 = 328 K

Calculate m_2, $m_2 = \dfrac{(170)(8.5)}{(0.1889)(328)} = 23.3 \text{ kg}$

Calculate m_{in}, m_{in} = 23.3 - 8.973 = 14.35 kg

Calculate u_2,

$$u_2 = \frac{1}{23.3}\left[(8.973)(154.49) + (14.35)(216.87) - 472.5\right] = 172.77 \text{ kJ/kg}$$

Lookup T_2 = 323 K

Repeat loop with T_2 = 323 K

Guess T_2 = 323 K

Calculate m_2, $m_2 = \dfrac{(170)(8.5)}{(0.1889)(323)} = 23.6 \text{ kg}$

Calculate m_{in}, m_{in} = 23.6 - 8.973 = 14.71 kg

Calculate u_2,

$$u_2 = \frac{1}{23.6}\left[(8.973)(154.49) + (14.71)(216.87) - 472.5\right] = 173.89 \text{ kJ/kg}$$

Lookup T_2 = 324 K

Stop loop since this should be close enough

2. A pressure cooker used in cooking has a relief valve that normally opens at about 180 kPa and a volume of 3 liters. Initially, 2/3 of the volume of the pressure cooker is filled with liquid. What is the highest heat transfer rate allowed for the pressure cooker not to run out of water in one hour of cooking?

Solution:

We begin by setting up our table.

System: Transient System

Working Fluid: water (compressible substance)

Process: Isobaric with P = 180 kPa

State 1: Fixed

State 2: Fixed

State Out: Fixed

$W_{sh} = 0$

Q: Unknown

$W_{bnd} = 0$

Conservation of mass: $m_2 - m_1 = -m_{out}$

$$1^{st} \text{ Law: } \frac{m_2 u_2 - m_1 u_1}{\Delta t} = -\dot{m}_{out} h_{out} + \dot{Q}$$

State 1	**State Out**	**State 2**
P_1 = 180 kPa	P_{out} = 180 kPa	P_2 = 180 kPa
T_1 = *116.9°C*	T_{out} = *116.9°C*	T_2 = *116.9°C*
V_1 = 0.003 m^3		V_2 = 0.003 m^3
m_1 = **1.8914 kg**	$\dot{m}_{out}$ = **5.245 x 10^{-4} kg/s**	m_2 = **0.0031 kg**
v_1 = **0.001586 m^3/kg**		v_2 = *0.9775 m^3/kg*
u_1 = *491.589 kJ/kg*	h_{out} = *2701.8 kJ/kg*	u_2 = *2525.9 kJ/kg*
Phase: 2 phase with x_1 = 5.287 x 10^{-4}	Phase: Sat. Vap.	Phase: Sat. Vap.

Italicized values are obtained from steam tables. **Bold** values are calculated.

We will assume that our mixture starts at 180 kPa. Then we need to determine the quality at state 1. The mass of vapor must be given by

$$m_{v,1} = \frac{(\text{\% vapor by volume})(\text{total volume})}{\text{specfic volume of vapor}} = \frac{(1/3)(0.003)}{v_g(@180 \text{ kPa})}$$

$$= \frac{(1/3)(0.003)}{0.9775} = 0.0010 \text{ kg}$$

Similarly for the mass of liquid we can write

$$m_{\ell,1} = \frac{\text{(\% liquid by volume)(total volume)}}{\text{specfic volume of liquid}} = \frac{(2/3)(0.003)}{v_f\,(@180\text{ kPa})}$$

$$= \frac{(2/3)(0.003)}{0.001058} = 1.8904 \text{ kg}$$

Then the total mass is

$$m_1 = m_{v,1} + m_{\ell,1} = 0.0010 + 1.8904 = 1.8914 \text{ kg}$$

and the quality and specific volume at state 1 are

$$x_1 = \frac{m_{v,1}}{m_1} = \frac{0.0010}{1.8914} = 5.287 \text{ x } 10^{-4}$$

$$v_1 = \frac{V_1}{m_1} = \frac{0.003}{1.8914} = 0.001586 \text{ m}^3/\text{kg}$$

The internal energy is then calculated from

$$u_1 = (1 - x_1)u_f + x_1 u_g$$

$$= (1 - 5.287 \text{ x } 10^{-4})(490.5) + (5.287 \text{ x } 10^{-4})(2525.9)$$

$$= 491.58 \text{ kJ/kg}$$

At state 2 we have saturated vapor, so the properties can be read directly form the steam tables. The mass at state 2 is determined from

$$m_2 = \frac{V_2}{v_2} = \frac{0.003}{0.9775} = 0.0031 \text{ kg}$$

The mass flow rate is calculated from the conservation of mass or

$$\dot{m}_{out} = \frac{m_1 - m_2}{\Delta t} = \frac{1.8914 - 0.0031}{3600} = 5.245 \text{ x } 10^{-4} \text{ kg}$$

The heat transfer rate is then determined from the 1st law or

$$\dot{Q} = \dot{m}_{out} h_{out} + \frac{m_2 u_2 - m_1 u_1}{\Delta t}$$

$$= (5.245 \text{ x } 10^{-4})(2701.8) + \frac{(0.0031)(2525.9) - (1.8914)(491.589)}{3600}$$

$$= 1.16 \text{ kW}$$

3. A 5 m^3 tank contains saturated liquid R-134a at 1.0 MPa. A small leak occurs and the tank pressure falls to 0.32 MPa before it is sealed. It is determined that the Freon remaining in the tank has a quality of 0.3. Determine

a. the mass that is discharged
b. the heat transfer that occurs

Solution:

We begin by setting up our table

System Type: Transient System
Substance Type: Compressible
Process: Isotropic with V = 5 m^3
State 1: Fixed
State 2: Fixed
State Out: Unknown
$W_{sh} = 0$
Q: Unknown
$W_{bnd} = 0$
Conservation of mass: $m_2 - m_1 = -m_{out}$

$$1^{st}\text{ Law: } \frac{m_2 u_2 - m_1 u_1}{\Delta t} = -\dot{m}_{out} h_{out} + \dot{Q}$$

State 1	**State Out**	**State 2**
P_1 = 1 MPa	P_{out} = **0.66 MPa**	P_2 = 0.32 MPa
T_1 = *39.39°C*	T_{out} = *24.7°C*	T_2 = *2.48°C*
V_1 = 5 m^3		V_2 = 5 m^3
m_1 = **5750 kg**	m_{out} =**5493.6 kg**	m_2 = **256.4 kg**
v_1 = *0.0008695 m^3/kg*		v_2 = *0.0195 m^3/kg*
u_1 = *104.42 kJ/kg*	h_{out} = *110.4 kJ/kg*	u_2 = *105.67 kJ/kg*
Phase: sat.liq.	Phase: two phase with x_{out} = 0.15	Phase: 2 phase with x=0.3

Italicized values are obtained from R-134a tables. **Bold** values are calculated.

At state 1 we have saturated liquid at 1 MPa, so we can go directly to the R-134a tables and find the properties. The mass at state 1 can then be calculated

$$m_1 = \frac{V_1}{v_1} = \frac{5}{0.0008695} = 5750\text{ kg}$$

At state 2 we know the pressure and the quality, so the properties can be determined. The mass at state 2 can then be calculated

$$m_2 = \frac{V_2}{v_2} = \frac{5}{0.0195} = 256.4\text{ kg}$$

The mass that is discharged now is calculated by the conservation of mass or

$m_{out} = m_1 - m_2 = 5750 - 256.4 = 5493.6$ kg

To fix state out, we use our average approach. We will assume that the outlet state is at the average pressure and the average quality then

$P_{out} = 0.66$ MPa and $x_{out} = 0.15$

So with the outlet properties determined we can use the 1st law to calculate the heat transfer or

$$\begin{aligned} Q &= m_{out}h_{out} + m_2u_2 - m_1u_1 \\ &= (5493.6)(110.4) + (256.4)(105.67) - (5750)(104.42) \\ &= 33{,}166 \text{ kJ} \end{aligned}$$