ME 201
Thermodynamics

Second Law Practice Problems

1. Ideally, which fluid can do more work: air at 600 psia and 600°F or steam at 600 psia and 600°F

Solution:

The maximum work a substance can do is given by its availablity. We will assume that we have a closed system so that

$$\psi = u - u_o - T_o(s - s_o)$$

We take the dead state to be at STP or 25°C and 100 kPa or 76.4°F and 14.7 psia. Then using the appropriate table we have

$$\psi_{air} = 183.30 - 91.53 - (537)\left(0.7649 - 0.5995 - 0.06855\ln\left[\frac{600}{14.7}\right]\right)$$

$$= 139.48 \text{ Btu / lb}_m$$

and

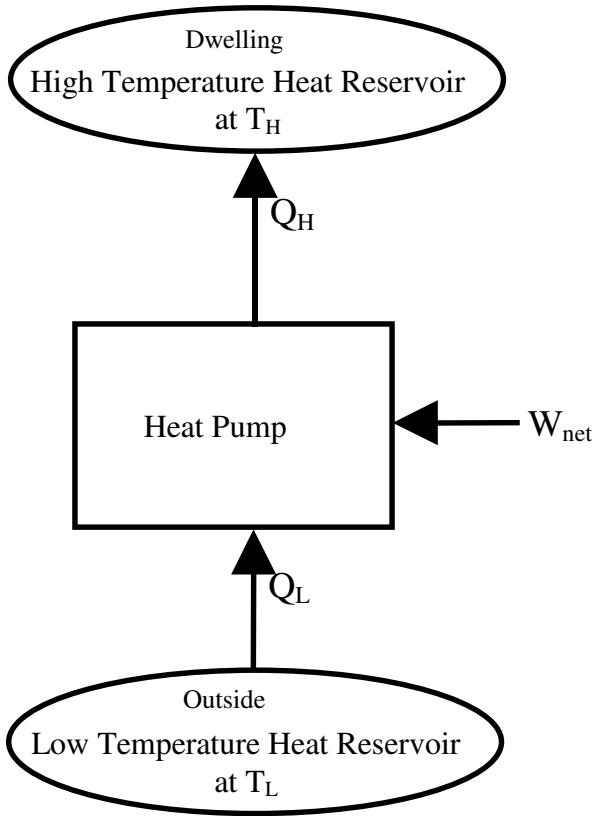
$$\psi_{steam} = 1184.5 - 44.09 - (537)(1.5320 - 0.08215)$$

$$= 361.84 \text{ Btu / lb}_m$$

So the steam can do more work

2. A heat pump provides 30,000 Btu/hr to maintain a dwelling at 68°F on a day when the outside temperature is 35°F. The power input to the pump is 1 hp. If electricity costs 8 cents per kilowatt-hour, compare the actual operating cost per day with the minimum theoretical operating cost per day.

Solution:
We sketch our device interactions



The cost is given by

$$\text{Cost} = (0.08)W_{net}$$

For the actual cost we have

$$(\text{Cost})_{act} = (0.08)(1)(0.7457\ \text{kW / hp})(24\ \text{hr / day})$$
$$= \$1.43$$

To calculate the minimum cost we will allow the heat pump to operate as a Carnot cycle, so that

$$COP_{Carnot} = \frac{1}{1-\frac{T_L}{T_H}} = \frac{1}{1-\frac{495}{528}} = 16$$

Then the minimum possible power input is

$$\left(\dot{W}_{net}\right)_{min} = \frac{\dot{Q}_H}{COP_{Carnot}} = \frac{30{,}000}{16} = 1875\ \text{Btu / hr}$$
$$= 0.5495\ \text{kW}$$

and the minimum cost is

$$(\text{Cost})_{\min} = (0.08)(0.5495)(24 \text{ hr / day})$$
$$= \$1.06$$

3. A cylinder/piston system contains water at 200 kPa, 200°C with a volume of 20 liters. The piston is moved slowly, compressing the water to a pressure of 800 kPa. The process is polytropic with a polytropic exponent of 1. Assuming that the room temperature is 20°C, show that this process does not violate the second law.

Solution:

To determine if this violate the 2nd law we will want to calculate the entropy change of the universe and compare it to zero. We have

$$(\Delta S)_{\text{universe}} = m(s_2 - s_1) + \frac{-Q_{sys}}{T_{surr}}$$

We now work this as a first law problem

Working Fluid: Water(compressible)
System: Closed System
Process: Polytropic with n=1.0

State 1	State 2
T_1 = 200°C	T_2 = *214.7°C*
P_1 = 200 kPa	P_2 = 800 kPa
u_1 = *2654.4 kJ/kg*	u_2 = *2655.5kJ/kg*
V_1 = 0.020 m^3	V_2 = **0.005 m^3**
v_1 = *1.0803 m^3/kg*	v_2 = **0.2703 m^3/kg**
s_1 = *7.5066 kJ/(kg K)*	s_2 = *6.8811 kJ/(kg K)*
phase: *sup.vap.*	phase: *sup.vap.*

italicized values from tables, **bold** values are calculated

Initial State: Fixed
Final State: Unknown
$W_{sh} = 0$
Q = ????
W_{bnd} = ????

We begin by calculating the mass

$$m = \frac{V_1}{v_1} = \frac{0.020}{1.0803} = 0.0185 \text{ kg}$$

To fix the final state we use the polytropic relationship

$$V_2 = \left[\frac{P_1 V_1^n}{P_2}\right]^{1/n} = \left[\frac{(200)(0.020)^1}{800}\right]^{1/1}$$

$$= 0.005 \text{ m}^3$$

The specific volume at state 2 is then

$$v_2 = \frac{V_2}{m} = \frac{0.005}{0.0185} = 0.2703 \text{ kg / m}^3$$

which gives us superheated vapor. The boundary work can be shown to be

$$W_{bnd} = P_1 V_1 \cdot \ln\left[\frac{V_2}{V_1}\right] = (200)(0.020)\ln\left(\frac{0.005}{0.020}\right)$$

$$= -5.55 \text{ kJ}$$

We use the first law to determine the heat transfer

$$Q_{sys} = m(u_2 - u_1) + W = (0.0185)(2655.5 - 2654.4) + (-5.55)$$

$$= -5.53 \text{ kJ}$$

Then

$$(\Delta S)_{surrounds} = \frac{-Q_{sys}}{T_{surr}} = \frac{-(-5.53)}{293} = 0.0189 \text{ kJ / K}$$

and for the system

$$(\Delta S)_{system} = m(s_2 - s_1) = (0.0185)(6.8811 - 7.5066)$$

$$= -0.0129 \text{ kJ / K}$$

So that

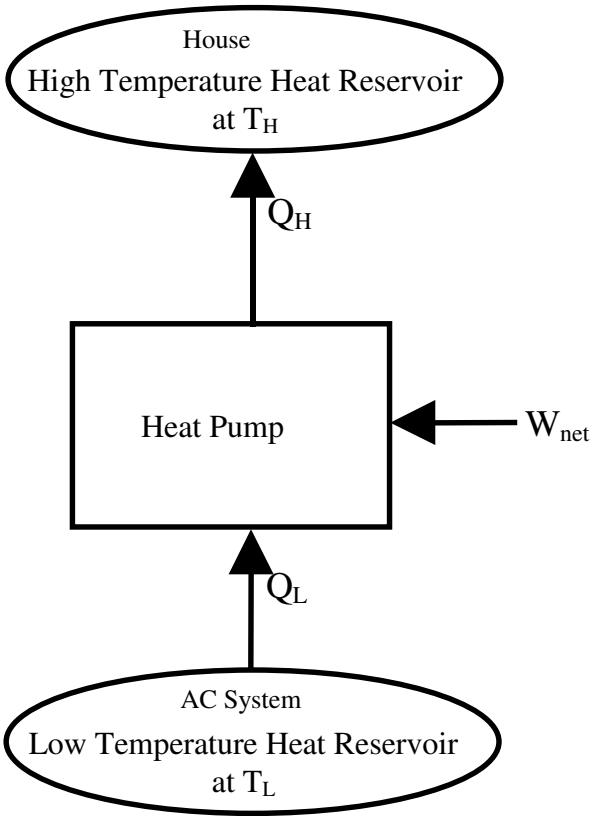
$$(\Delta S)_{universe} = -0.0129 + 0.0189 = 0.006 \text{ kJ / K}$$

Since this is greater than zero the second law is not violated.

4. When a man returns to his well-sealed house on a summer day, he finds that the house is at 32°C. He turns on the air conditioner which cools the entire house to 20°C in 15 minutes. If the COP of the heat pump system is 2.5, determine the power drawn by the heat pump. Assume the entire mass within the house is equivalent to 800 kg of air.

Solution:

We begin by sketching our device interactions



By definition we have

$$COP = \frac{\dot{Q}_H}{\dot{W}_{net}}$$

So if the required heat transfer can be determined the power can be determined. From a first law analysis on the house, we can write

$$\dot{Q} = m\frac{u_2 - u_1}{t} = (800)\frac{209.06 - 217.67}{(15)(60)}$$

$$= -7.65 \text{ kW}$$

and

$$\dot{Q}_H = -\dot{Q} = 7.65 \text{ kW}$$

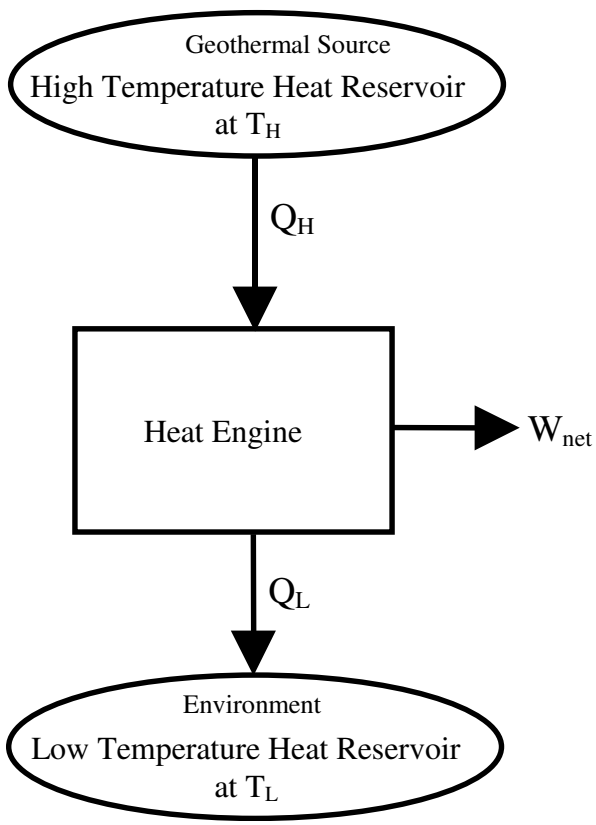
Then the power required is

$$\dot{W}_{net} = \frac{\dot{Q}_H}{COP} = \frac{7.65}{2.5} = 3.06 \text{ kW}$$

5. An innovative way of power generation involves the utilization of geothermal energy, the energy of hot water that exists naturally underground (hot springs), as the heat source. If a supply of hot water at 140°C is discovered at a location where the environmental temperature is 20°C, determine the maximum thermal efficiency a geothermal plant built at that location can have. If the power output of the plant is to be 5 MW, what is the minimum mass flow rate of hot water needed?

Solution:

We begin by sketching our device interactions



The maximum thermal efficiency will occur when the heat engine operates as a Carnot cycle,

$$\eta_{th} = \eta_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{(20+273)}{(140+273)} = 0.291$$

The minimum mass flow rate of hot water corresponds to the maximum thermal efficiency or

$$\left(\dot{Q}_H\right)_{min} = \frac{\dot{W}_{net}}{\eta_{Carnot}} = \frac{5000}{0.291} = 17{,}208 \text{ kW}$$

Performing a first law analysis on the hot water stream we have

$$Q = \dot{m}(h_{out} - h_{in})$$

For the minimum flow rate we will assume that the hot water is cooled down to the environment temperature, then

$$\dot{m} = \frac{Q}{c_P(T_{out} - T_{in})} = \frac{-17{,}208}{(4.1978)(20-140)}$$

$$= 34.2 \text{ kg/s}$$

6. Air enters an adiabatic non-ideal nozzle at 9 m/s, 300 K, and 120 kPa and exits at 100 m/s and 100 kPa. Determine the irreversibility and the reversible work on a per mass basis.

Solution:

We first solve this as a first law problem

Working Fluid: Air(ideal gas)
System: Control Volume System
Process: Nozzle

State 1	State 2
$T_1 = 300K$	$T_2 = 295°C$
$P_1 = 120$ kPa	$P_2 = 100$ kPa
$h_1 =$ *300.19 kJ/kg*	$h_2 =$ **295.04 kJ/kg**
$\phi_1 =$ *1.70203 kJ/(kg K)*	$\phi_2 =$ *1.68515 kJ/(kg K)*
$\vec{v}_1 = 9$ m/s	$\vec{v}_2 = 100$ m/s

italicized values from tables, **bold** values are calculated

Initial State: Fixed
Final State: ???
$W_{sh} = 0$
$Q = 0$

We use the first law to fix the final state

$$h_1 + \frac{\vec{v}_1}{2} = h_2 + \frac{\vec{v}_2}{2}$$

Then solving for h_2

$$h_2 = h_1 + \frac{\vec{v}_1^2 - \vec{v}_2^2}{2} = 300.19 + \frac{(9)^2 - (100)^2}{2}(10^{-3})$$

$$= 295.04 \text{ kJ/kg}$$

which allows us to determine the temperature and ϕ_2. Then the reversible work is

$$w_{rev} = h_1 - h_2 - T_{HR}\left(\phi_1 - \phi_2 - R \cdot \ln\left[\frac{P_1}{P_2}\right]\right)$$

$$= 300.19 - 295.04 - (298)\left(1.70203 - 1.68515 - (0.287)\ln\left[\frac{120}{100}\right]\right)$$

$$= 15.71 \text{ kJ / kg}$$

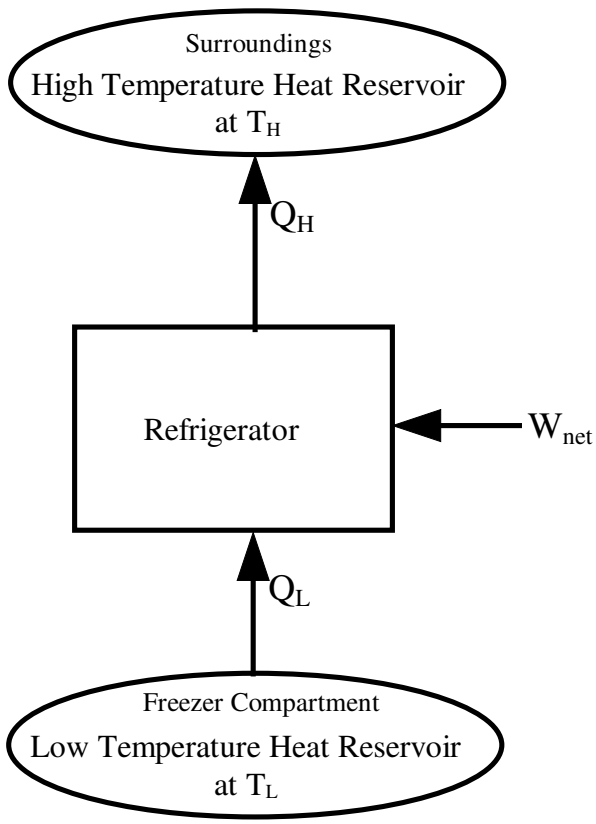
Since the actual work is zero the irreversibility is

$$i = w_{rev} = 15.71 \text{ kJ / kg}$$

7. Determine if a tray of ice cubes could remain frozen when placed in a food freezer having a COP of 9, operating in a room where the temperature is 32°C.

Solution:

We begin by sketching our device interactions



Assuming that the refrigerator operates on the Carnot cycle, we have

$$COP = COP_{Carnot} = \frac{1}{\frac{T_H}{T_L} - 1}$$

Solving for T_L

$$T_L = \frac{T_H}{\frac{1}{COP} + 1} = \frac{305}{\frac{1}{9} + 1} = 274.5 \text{ K}$$

and since this is greater than 0°C the ice cubes will not remain frozen.

8. Air is compressed in a closed system from a state where the pressure is 100 kPa and the temperature is 27°C to a final state at 500 kPa and 177°C. Can this process occur adiabatically? If yes, determine the work per mass. If no, determine the direction of the heat transfer.

Solution:

To determine if the process can occur, we must calculate

$$(\Delta S)_{universe} = m(s_2 - s_1) + \frac{-Q_{sys}}{T_{surr}}$$

and compare it to zero. Since the process is adiabatic

$$(\Delta s)_{universe} = s_2 - s_1 = \phi_2 - \phi_1 - R \cdot \ln\left(\frac{P_2}{P_1}\right)$$

Going to the air tables we find

$$(\Delta s)_{universe} = s_2 - s_1 = 2.11161 - 1.70203 - (0.287)\ln\left(\frac{500}{100}\right)$$

$$= -0.523 \text{ kJ / kg}$$

Since this is less than zero, the process cannot be adiabatic. To make $\Delta s_{universe}$ greater than zero will require Q_{sys} to be negative, so that the direction of heat transfer is out of the system.

9. The pressure of water is increased by the use of a pump from 14 to 40 psia. A rise in the water temperature from 60°F to 60.2°F is observed. Determine the irreversibility, the second law efficiency, and the isentropic efficiency of the pump.

Solution:

We first solve this as a first law problem

Working Fluid: Water (incompressible)
System: Control Volume System
Process: Pump

State 1	State 2s (ideal)	State 2a (actual
$T_1 = 60°F$	$T_{2s} =$	$T_{2a} = 60.2°F$
$P_1 = 14$ psia	$P_2 = 40$ psia	$P_2 = 40$ psia

bold values are calculated
Initial State: Fixed
Final State: fixed
$W_{sh} = ????$
$Q = 0$

To calculate the irreversibility, we use

$$\begin{aligned} i &= T_{HR}(s_2 - s_1) - q \\ &= (537)c_{p,avg}\ln\left(\frac{T_2}{T_1}\right) - 0 \\ &= (537)(1.0014)\ln\left(\frac{520.2}{520}\right) \\ &= 0.2068 \text{ Btu / lb}_m \end{aligned}$$

To determine the second law efficiency we need both the actual work and the ideal work. Starting with the actual work we have

$$\begin{aligned} w_{act} &= h_1 - h_2 + q \\ &= c_{p,avg}(T_1 - T_2) + v_{avg}(P_1 - P_2) - 0 \\ &= (1.0014)(60 - 60.2) + (0.016035)(14 - 40) / (5.40395 \text{ psia} \cdot \text{ft}^3 / \text{Btu}) \\ &= -0.2774 \text{ Btu / lb}_m \end{aligned}$$

The reversible work is given by

$$\begin{aligned} w_{rev} &= i + w_{act} = (0.2068) + (-0.2774) \\ &= -0.0706 \text{ Btu / lb}_m \end{aligned}$$

which allows us to determine the second law efficiency as

$$\eta_{II} = \frac{w_{rev}}{w_{act}} = \frac{(-0.0706)}{(-0.2774)} = 0.255$$

To determine the isentropic efficiency, we must first calculate the ideal work. Recognizing that in a isentropic process, the water will not change temperature, we can write

$$w_{ideal} = v_{avg}(P_1 - P_2)$$

$$= (0.016035)(14 - 40) / (5.40395 \text{ psia} \cdot \text{ft}^3 / \text{Btu})$$

$$= -0.0771 \text{ Btu} / \text{lb}_m$$

Then our isentropic efficiency is

$$\eta_s = \frac{w_{ideal}}{w_{act}} = \frac{(-0.0771)}{(-0.2774)} = 0.2781$$

10. Carbon dioxide undergoes an isothermal reversible process from 250 kPa and 300°C to 500 kPa. Determine the heat transfer per mass by using the first law and evaluating the boundary work from $\int Pdv$. Compare this to the heat transfer per mass calculated from the entropy change and the second law.

Solution:

We first solve this as a first law problem

Working Fluid: CO_2 (ideal gas)
System: Closed System
Process: Isothermal, Reversible

State 1	State 2
T_1 = 300C = 573K	$T_2 = T_1$ = 573K
P_1 = 250 kPa	P_2 = 500 kPa
u_1 = *369.23 kJ/kg*	u_2 = *369.23 kJ/kg*
ϕ_1 = *5.478 kJ/(kg K)*	ϕ_2 = *5.478 kJ/(kg K)*
v_1 = *0.433* m^3*/kg*	v_2 = *0.2165* m^3*/kg*

italicized values are from ideal gas relationships

Initial State: Fixed
Final State: fixed

$W_{sh} = 0$
Q = ???
W_{bnd} = ????

We first go to the CO_2 tables and get our properties. Our boundary work for an ideal gas undergoing an isothermal process is

$$w_{bnd} = RT \cdot \ln\left(\frac{v_2}{v_1}\right)$$

$$= (0.1889)(573)\ln\left(\frac{0.2165}{0.433}\right)$$

$$= -75.03 \text{ Btu / lb}_m$$

Using the first law our heat transfer is

$$q = u_2 - u_1 + w_{bnd}$$

$$= 369.23 - 369.23 + (-75.036)$$

$$= -75.036 \text{ Btu / lb}_m$$

From the second law we have

$$q = T_1(s_2 - s_1)$$

$$= T_1\left(\phi_2 - \phi_1 - R \cdot \ln\left[\frac{P_2}{P_1}\right]\right)$$

$$= (573)\left(5.478 - 5.478 - (0.1889)\ln\left[\frac{500}{250}\right]\right)$$

$$= -75.03 \text{ Btu / lb}_m$$

So the two calculations for heat transfer agree.