ME 201
Thermodynamics

Old Final Exam Answers

Directions: Open book, open notes. Work all four problems. Problems are equally weighted.

Problem 1

Consider applying our Carnot heat engine approach to a biological system, specifically, a hunting cheetah. The work output may be considered to be the kinetic energy change as the 45 kg cheetah accelerates from rest to 27 m/s. The heat input, Q_H, may be considered to come from the digestion of meat. The outside temperature is 30°C and the cheetah's internal temperature from which the Q_H comes from is 42°C. Determine

a. the heat that must be supplied
b. the heat that must be rejected
c. the kilograms of meat that must be digested if 200 kJ of heat is released for every kilogram of meat digested

Answers:

(a) The heat transfer from the high temperature heat reservoir is

$$Q_H = \frac{W_{net}}{\eta} = \frac{16.403}{0.0381} = 430.58\text{ kJ}$$

(b) The heat transfer from the low temperature heat reservoir is

$$Q_L = Q_H - W_{net} = 430.58 - 16.403 = 414.18\text{ kJ}$$

(c) The mass of meat that need to be consumed (and digested) is

$$m_{meat} = \frac{Q_H}{e_{meat}} = \frac{430.58}{200} = 2.15\text{ kg}$$

Problem 2

Steam at 8 MPa, 400°C, and 50 kg/s enters a turbine with isentropic efficiency of 78% and exhausts to a pressure of 40 kPa. Determine

a. actual power output of the turbine
b. second law efficiency for the turbine
c. difference in steam exit quality between ideal and actual operation

Answers:

State 1 (inlet)	State 2s (ideal outlet)	State 2a (actual outlet)
T_1 = 400°C	T_{2s} = *75.87°C*	T_{2a} = *75.87°C*
P_1 = 8 MPa	P_{2s} = 40 kPa	P_{2a} = 40 kPa
h_1 = *3139.4 kJ/kg*	h_{2s} = *2181.23 kJ/kg*	h_{2a} = **2392 kJ/kg**
$\dot{m}_1 = 50\text{ kg/s}$	$\dot{m}_{2s} = 50\text{ kg/s}$	$\dot{m}_{2a} = 50\text{ kg/s}$
s_1 = *6.3658 kJ/(kg K)*	s_{2s} = **6.3658 kJ/(kg K)**	s_{2a} = *6.9697 kJ/(kg K)*
phase: *sup.vap*	phase: *2 phase with* $x_{2s} = 0.804$	phase: *2 phase with* $x_{2a} = 0.895$

Italicized values from tables. **Bold** values are calculated

(a) The actual power is calculated from the efficiency

$$\dot{W}_{act} = \eta_s \dot{W}_{ideal} = (0.78)(47{,}909) = 37{,}368 \text{ kW}$$

(b) The second law efficiency is

$$\eta_{II} = \frac{37{,}368}{46{,}368} = 0.806$$

(c) The qualities are

$x_{2s} = 0.804$

$x_{2a} = 0.895$

Problem 3

Consider a steam power cycle with double reheating as shown in Figure 1. Steam enters the first turbine at 25 MPa and 600°C. Both reheat legs produce steam at 600°C and the exhaust pressure of the third and final turbine is 10 kPa. For plant optimization the exit state for all three turbines is saturated vapor. Determine

a. operating pressures for the two reheat legs
b. heat supplied during the two reheat legs
c. work output by the three turbines

Answers:

Node	T(°C)	P(kPa)	Fluid Phase	h(kJ/kg)	s(kJ/kg·K)
1	600	25000	*sup.vap*	*3493.5*	*6.3637*
2	*209*	*1870.7*	sat.vap	*2796.8*	**6.3637**
3	600	**1870.7**	*sup.vap.*	*3691.73*	*7.7366*
4	*71.2*	*33.14*	sat.vap	*2628.21*	**7.7366**
5	600	**33.14**	*sup.vap.*	*3706.13*	*9.7333*
6	*45.81*	10	sat.vap.	*2583.9*	*8.1488*

Italicized values are from steam tables. **Bold** values are calculated

$$\begin{aligned} w_{turbines} &= (h_1 - h_2) + (h_3 - h_4) + (h_5 - h_6) \\ &= (3493.5 - 2769.8) + (3691.73 - 2628.21) + (3706.13 - 2583.9) \\ &= 2909.5 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} q_{reheat} &= (h_3 - h_2) + (h_5 - h_4) \\ &= (3691.73 - 2769.8) + (3705.3 - 2628.21) \\ &= 1999 \text{ kJ/kg} \end{aligned}$$

Problem 4

Somerton Engineering has developed the new Rhino Engine, a six cylinder internal combustion engine based upon the Otto cycle. However, in the Rhino Engine the power stroke can be modeled as an isentropic expansion from the top dead center volume to 80% of the bottom dead center volume followed by a polytropic expansion (with polytropic exponent of 1.2) to bottom dead center (see Fig. 2). The engine has the following operating characteristics

Displacement volume (for all 6 cylinders): 2.4 liters
Compression ratio: 9.5
Temperature at start of power stroke: 2200 K
Pressure at start of power stroke: 9.44 MPa

The temperature and pressure at the start of the cycle are 310 K and 140 kPa.

Determine

a. temperature and pressure at end of power stroke
b. work output from only the power stroke
c. heat transfer in the polytropic process

Solution:

$(V_{disp})_{cylinder} = V_{disp}/6 = 4 \times 10^{-4}\ m^3$

$$\left(V_{disp}\right)_{cylinder} = V_{BDC} - V_{TDC}$$

$$V_{BDC} = rV_{TDC}$$

$$V_{TDC} = \frac{V_{disp}}{r-1} = \frac{4 \times 10^{-4}}{9.5-1} = 4.7059 \times 10^{-5}\ m^3$$

$$V_{BDC} = (9.5)(4.7059 \times 10^{-5}) = 4.4706 \times 10^{-4}\,m^3$$

$$V_{int} = 0.8V_{BDC} = (0.8)(4.4706 \times 10^{-3}) = 3.5765 \times 10^{-4}\,m^3$$

Node	T(K)	P(kPa)	V(m^3)	v(m^3/kg)	u(kJ/kg)	v_r
1	2200	9,440	4.7059 x 10^{-5}	*0.0669*	*1872.4*	*2.012*
2	*1178.9*	*665.6*	3.5765 x 10^{-4}	**0.5084**	*914.6*	**15.287**
3	*1127.4*	**509.2**	4.4706 x 10^{-4}	**0.6355**	*869.3*	

Italicized values from ideal gas relations. **Bold** values calculated.

$$m = \frac{V_{TDC}}{v_1} = \frac{4.7059 \times 10^{-5}}{0.0669} = 7.0342 \times 10^{-4}\ \text{kg}$$

$$W_{1-2} = m\left(u_1 - u_2\right) = (7.0342 \times 10^{-4})(1872.4 - 914.6) = 0.6739\ \text{kJ}$$

$$W_{2-3} = \frac{P_3V_3 - P_2V_2}{1-n} = \frac{(509.2)(4.4706 \times 10^{-4}) - (665.6)(3.5765 \times 10^{-4})}{1-1.2}$$

$$= 0.0520\ \text{kJ}$$

$W_{tot} = 0.7259$ kJ

$$Q_{2-3} = m\left(u_3 - u_2\right) + W_{2-3} = (7.0342 \times 10^{-4})(869.3 - 914.6) + 0.0520$$

$$= 0.0201\ \text{kJ}$$