ME 201
Thermodynamics

Old Exam #2 Solutions

Directions: Work all three (3) problems. The exam is open notes and open text book. All problems have equal weight.

Problem 1
A rigid container holds 3 kg of air initially at 50°C. The air is stirred so that its pressure changes from 500 kPa to 2000 kPa. The heat transfer is 200 kJ. Determine the final temperature, the change in internal energy, and the work.

Solution:
Using our 1st law template, we carefully read the problem and enter the given information.

<table>
<thead>
<tr>
<th>State Type</th>
<th>System Type: Closed (container)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Substance Type</td>
<td>Substance Type: Ideal Gas (air)</td>
</tr>
<tr>
<td>Process Type</td>
<td>Process Type: Isotropic (rigid)</td>
</tr>
<tr>
<td>State 1</td>
<td>State 1: Fixed</td>
</tr>
<tr>
<td>T₁ = 50°C = 323 K</td>
<td>T₂ = 1292 K</td>
</tr>
<tr>
<td>P₁ = 500 kPa</td>
<td>P₂ = 2000 kPa</td>
</tr>
<tr>
<td>V₁ = 0.1854 m³/kg</td>
<td>V₂ = 0.1854 m³/kg</td>
</tr>
<tr>
<td>u₁ = 230.58 kJ/kg</td>
<td>u₂ = 1015.6 kJ/kg</td>
</tr>
<tr>
<td>m₁ = 3 kg</td>
<td>m₂ = 3 kg</td>
</tr>
</tbody>
</table>

Bold values are calculated. Italicized values are from tables or ideal gas law.

Conservation of Energy:

\[ m₁(u₂ - u₁) = Q - W_{sh} \]

From the air tables we find

\[ u₁ = 230.58 \text{ kJ/kg} \]

From the ideal gas law we have

\[ v₁ = \frac{RT₁}{P₁} = \frac{(0.287)(323)}{(500)} = 0.1854 \text{ m}³/\text{kg} \]
Then by definition
\[ V_1 = m v_1 = (3)(0.1854) = 0.5562 \text{ m}^3 \]
Since our process is isotropic
\[ V_2 = V_1 = 0.5562 \text{ m}^3 \]
and
\[ v_2 = v_1 = 0.1854 \text{ m}^3/\text{kg} \]
Solving for the state 2 temperature from the ideal gas law
\[ T_1 = \frac{P_1 v_1}{R} = \frac{(2000)(0.1854)}{(0.287)} = 1292 \text{ K} \]
From the air tables we now find
\[ u_2 = 1015.6 \text{ kJ/kg} \]
Solving for the shaft work form the 1st law
\[ W_{sh} = Q - m_1 (u_2 - u_1) = 200 - (3)(1015.6 - 230.58) = -2155 \text{ kJ} \]

**Problem 2**
A desuperheater is an adiabatic mixing tank which produces saturated vapor from a superheated vapor by adding liquid. If superheated steam at 3.5 MPa, 400°C enters at the rate of 0.5 kg/s, and liquid water enters the unit at 3.5 MPa and 40°C, determine the amount of saturated vapor at 3.0 MPa which can be produced. What is the flow rate of liquid water required?

**Solution:**

System Type: Control Volume
Substance Type: Compressible (steam)
Device: Mixing Tank(isobaric and adiabatic)
State 1 (water inlet): Fixed
State 2 (steam inlet): Fixed
State 3 (steam outlet): unknown
\[ W_{sh} = 0 \]
\[ Q = 0 \]
conservation of mass: \[ m_1 + m_2 = m_3 \]
1st law: \[ m_1 h_1 + m_2 h_2 = m_3 h_3 \]

<table>
<thead>
<tr>
<th>State 1 (water inlet)</th>
<th>State 2 (steam inlet)</th>
<th>State 3 (steam outlet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_1 = 40^\circ \text{C} )</td>
<td>( T_2 = 400^\circ \text{C} )</td>
<td>( T_3 = 242.56^\circ \text{C} )</td>
</tr>
<tr>
<td>( P_1 = 3.5 \text{ MPa} )</td>
<td>( P_2 = 3.5 \text{ MPa} )</td>
<td>( P_3 = 3.5 \text{ MPa} )</td>
</tr>
<tr>
<td>( h_1 = 171.05 \text{ kJ/kg} )</td>
<td>( h_2 = 3223.2 \text{ kJ/kg} )</td>
<td>( h_3 = 2802.7 \text{ kJ/kg} )</td>
</tr>
<tr>
<td>( \dot{m}_1 = 0.0799 \text{ kg/s} )</td>
<td>( \dot{m}_2 = 0.5 \text{ kg/s} )</td>
<td>( \dot{m}_3 = 0.5799 \text{ kg/s} )</td>
</tr>
</tbody>
</table>

*Italicized* values from tables. **Bold** values are calculated

Approach: Since state 1 and state 2 are fixed we may go to the steam tables and obtain the remaining properties. State 3 is fixed from the isobaric condition, and the remaining properties for state 3 are then obtained from the steam tables. The conservation of mass and 1st law must be used in conjunction to determine the mass flow rates.

At state 1 we can go to the saturation pressure table and find that at 3.5Pa, the saturation temperature is 242.56°C, so that we have subcooled liquid. Unfortunately, the pressure is not
high enough to use the compressed liquid tables, so we will have to model this as an incompressible substance, but use the saturation properties. Then
\[ h = h_f(at\ T_{given}) + v_f(at\ T_{given})[P_{given} - P_{sat}(at\ T_{given})] \]
Then using the steam tables
\[ h_1 = 167.53 + (0.001008)(3500 - 7.3851) = 171.05 \text{ kJ/kg} \]
For state 2 we find that at 3.5 MPa the saturation temperature is 242.56°C, so that we have superheated vapor. Then from the superheat table we find
\[ h_2 = 322.32 \text{ kJ/kg} \]
Then at state 3, we know that the mixing tank is isobaric, so that
\[ \bar{P}_3 = \bar{P}_1 = 3.5 \text{ MPa} \]
Then for saturated vapor at 3.5 MPa
\[ T_3 = 242.56\degree C \text{ and } h_3 = 2802.7 \text{ kJ/kg} \]
Using the conservation of mass we find
\[ \bar{m}_1 = \bar{m}_1 + \bar{m}_2 \]
Substituting for \( \bar{m}_1 \) in our 1\textsuperscript{st} law gives
\[ \bar{m}_1 h_1 + \bar{m}_2 h_2 = (\bar{m}_1 + \bar{m}_2) h_3 \]
Solving the first law for \( \bar{m}_1 \) gives
\[ \bar{m}_1 = \frac{\bar{m}_1 h_3 - \bar{m}_2 h_2}{h_1 - h_3} = \frac{(0.5)(2802.7 - 322.32)}{(171.05 - 2802.7)} = 0.0799 \text{ kg/s} \]
and for the mass flow rate at 3
\[ \bar{m}_3 = 0.0799 + 0.5 = 0.5799 \text{ kg/s} \]

Problem 3
What temperature must air be at prior to entering an ideal nozzle if an exit velocity of 1200 m/sec is required? The nozzle inlet pressure is 400 kPa, the inlet kinetic energy is negligible, and the outlet temperature is 1500 K. What is the exit pressure?

**Solution:**

System: Control Volume
Substance: Air (Ideal Gas)
Device: Nozzle (isentropic, flow device)
Inlet State: Not Fixed
Outlet State: Not Fixed
Q = 0
W = 0

1\textsuperscript{st} Law: \[ h_2 + \frac{v_2^2}{2} - h_1 = 0 \]

<table>
<thead>
<tr>
<th>State 1</th>
<th>State 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_1 = 2082.8 \text{ K} )</td>
<td>( T_2 = 1500 \text{ K} )</td>
</tr>
<tr>
<td>( P_1 = 400 \text{ kPa} )</td>
<td>( P_2 = 97.6 \text{ kPa} )</td>
</tr>
<tr>
<td>( h_1 = 2356 \text{ kJ/kg} )</td>
<td>( h_2 = 1635.97 \text{ kJ/kg} )</td>
</tr>
<tr>
<td>( s_{v1} = 3.8501 \text{ kJ/(kg} \cdot \text{K}) )</td>
<td>( s_{v2} = 3.44516 \text{ kJ/(kg} \cdot \text{K}) )</td>
</tr>
<tr>
<td>( v_1 = 0 \text{ m/s} )</td>
<td>( v_2 = 1200 \text{ m/s} )</td>
</tr>
</tbody>
</table>

**Bold** values calculated. **Italicized** values from air tables
Approach: We can use the first law to solve for $h_1$. The air tables can then be used to give us $T_1$. The isentropic condition is then used to calculate $P_2$.

For state 2 we go to the air tables and find

$h_2 = 1635.97 \text{ kJ/kg}$ and $s^o_2 = 3.44516 \text{ kJ/(kg\cdot K)}$

Our 1st law is written

$h_2 - h_1 + \frac{\bar{v}_2^2}{2} = 0$

Solving for $h_2$

$h_1 = h_2 + \frac{\bar{v}_2^2}{2} = 1635.97 + \frac{(1200)^2 \cdot 10^{-3}}{2} = 2356 \text{ kJ/kg}$

Going to the air tables we find

$T_1 = 2082.8 \text{ K}$ and $s^o_1 = 3.8501 \text{ kJ/(kg\cdot K)}$

Our isentropic relationship is

$0 = s^o_2 - s^o_1 - R \cdot \ln \left( \frac{P_2}{P_1} \right)$

Solving for $P_2$

$P_2 = P_1 \cdot \exp \left( \frac{s^o_2 - s^o_1}{R} \right) = (400) \exp \left( \frac{3.44516 - 3.8501}{0.287} \right) = 97.6 \text{ kPa}$