ME 201
Thermodynamics

Conservation of Mass Practice Problems

1. A human being can blow air out of their mouth at a rate of 10^{-4} kg/s. How long will it take for this human to blow up a balloon to a volume of 5 x 10^{-4} m^3? The air may be taken to be at 311 K and 110 kPa.

Solution: This is a conservation of mass problem with one inflow and no outflows and zero initial mass. Writing the conservation of mass, we have

$$\frac{m_{final} - m_{initial}}{\Delta t} = \dot{m}_{in}$$

Assuming that the initial mass is zero and solving for the time period, we have

$$\Delta t = \frac{m_{final}}{\dot{m}_{in}}$$

For the final mass, we have

$$m_{final} = \frac{V_{final}}{v}$$

We will determine the specific volume, v, for air from the ideal gas equations, or

$$v = \frac{RT}{P} = \frac{(0.287)(311)}{(110)} = 0.81\,m^3/kg$$

Then

$$m_{final} = \frac{5\,x\,10^{-4}}{0.81} = 6.17\,x\,10^{-4}\ kg$$

So that

$$\Delta t = \frac{6.17\,x\,10^{-4}}{10^{-4}} = 6.2\,s$$

2. Consider a 40 gallon hot water heater. Over a fifteen minute time period hot water flows out of the hot water heater at 0.75 kg/s and cold water at 0.5 kg/s flows into the hot water heater. How full (in percent) is the hot water heater at the end of fifteen minutes? You may take the water temperature to be 85ºC.

Solution: This is a conservation of mass problem with one inflow and one outflow with mass depletion. Writing the conservation of mass, we have

$$\frac{m_{final} - m_{initial}}{\Delta t} = \dot{m}_{in} - \dot{m}_{out}$$

We want to determine the final mass which will then allows us to determine how full the hot water is. Solving for the final mass, we have

$$m_{final} = m_{initial} + \Delta t(\dot{m}_{in} - \dot{m}_{out})$$

The time period is given as 15 minutes or 900 seconds. The initial mass is given by

$$m_{initial} = \rho V_{initial}$$

The initial volume is 80 gallons or 0.30 cubic meters. We take the density for liquid water at 85ºC or interpolating form Table A-3, 968.2 kg/m^3. Now solving

$$m_{initial} = (968.2)(0.30) = 290.5\,\text{kg}$$

Then

$$m_{final} = 290.5 + (900)(0.50 - 0.75) = 65.5\,\text{kg}$$

So at the end of 15 minutes the tank is 23% full.

3. If it takes 12 seconds to empty a one liter bottle, what is the mass flow rate of soda leaving the bottle? You may take the density of soda to be 850 kg/s.

Solution: This is a conservation of mass problem with one outflow and no inflows and zero final mass. Writing the conservation of mass, we have

$$\frac{-m_{initial}}{\Delta t} = -\dot{m}_{out}$$

Solving we have

$$\dot{m}_{out} = \frac{m_{initial}}{\Delta t} = \frac{\rho V_{initial}}{\Delta t} = \frac{(850)(1/1000)}{12} = 0.071\,\text{kg/s}$$