

ME 201

Thermodynamics

Ideal Gas Property Evaluation Guide (For ME 201 see summary at end)

Most normal gases at normal pressures and temperature can be treated as ideal gases provided that there are not phase changes occurring.

The equation of state for an ideal gas provides a relationship among temperature (T), pressure (P), and the specific volume (v). The most convenient form of the ideal gas equation of state is

$$Pv = RT$$

where R is the gas constant for the particular of gas of interest and is given by

$$R = \frac{R_u}{MW}$$

where MW is the molecular weight of the particular gas and R_u is the universal gas constant. Some useful values of the universal gas constant are given below:

$$\begin{aligned}R_u &= 8.314 \text{ kJ} / (\text{kgmole} \cdot \text{K}) \\&= 8.314 \text{ kPa} \cdot \text{m}^3 / (\text{kgmole} \cdot \text{K}) \\&= 1.986 \text{ Btu} / (\text{lb}_m \text{mole} \cdot \text{R}) \\&= 10.73 \text{ psia} \cdot \text{ft}^3 / (\text{lb}_m \text{mole} \cdot \text{R})\end{aligned}$$

For an ideal gas we also a unique relationship among the two specific heats and the gas constant, as shown below

$$c_v = c_p - R$$

Sometimes it is convenient to use the ratio of the specific heats, k , defined as

$$k = \frac{c_p}{c_v}$$

There are three ways to evaluate changes in internal energy, enthalpy, and entropy for an ideal gas.

(i) Constant specific heat

Assuming a constant specific heat gives

$$u_2 - u_1 = c_{v,avg}(T_2 - T_1)$$

$$h_2 - h_1 = c_{p,avg}(T_2 - T_1)$$

$$s_2 - s_1 = c_{p,avg} \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{P_2}{P_1}\right)$$

where

$$c_{p,avg} = c_p\left(@\frac{T_1 + T_2}{2}\right) \text{ and}$$

$$c_{v,avg} = c_v\left(@\frac{T_1 + T_2}{2}\right)$$

(ii) Assume polynomial relationship for specific heat;

Quite often a polynomial expression for the specific heat as a function of temperature is provided. In the text by Potter and Somerton, a cubic relationship is given for the specific heat on a per mole basis. Converting to a per mass basis, we can write.

$$c_p = \frac{1}{MW}(a + b \cdot T + c \cdot T^2 + d \cdot T^3)$$

Using this in our integrals gives

$$\begin{aligned}
 u_2 - u_1 &= \frac{1}{MW} \left[(a - R_u) \cdot (T_2 - T_1) + \frac{b}{2} \cdot (T_2^2 - T_1^2) \right. \\
 &\quad \left. + \frac{c}{3} \cdot (T_2^3 - T_1^3) + \frac{d}{4} \cdot (T_2^4 - T_1^4) \right] \\
 h_2 - h_1 &= \frac{1}{MW} \left[a \cdot (T_2 - T_1) + \frac{b}{2} \cdot (T_2^2 - T_1^2) \right. \\
 &\quad \left. + \frac{c}{3} \cdot (T_2^3 - T_1^3) + \frac{d}{4} \cdot (T_2^4 - T_1^4) \right] \\
 s_2 - s_1 &= \frac{1}{MW} \left[a \cdot \ln\left(\frac{T_2}{T_1}\right) + b \cdot (T_2 - T_1) \right. \\
 &\quad \left. + \frac{c}{2} \cdot (T_2^2 - T_1^2) + \frac{d}{3} \cdot (T_2^3 - T_1^3) \right] - R \cdot \ln\left(\frac{P_2}{P_1}\right)
 \end{aligned}$$

(iii) Using the ideal gas tables.

With the temperature known, the internal energy (u) and the enthalpy (h) can be read directly from the table. The temperature part of the entropy, denoted by ϕ in our handout but by s° in the textbook, is also read from the table. To determine the entropy change, we must include the pressure part so that

$$s_2 - s_1 = \phi_2 - \phi_1 - R \cdot \ln\left(\frac{P_2}{P_1}\right)$$

When using ideal gas tables for gases other than air, note that the values are on a per mole basis.

Summary for ME 201 Students Relationships for an Ideal Gas

$$Pv = RT$$

$$c_v = c_p - R$$

Assuming Constant Specific Heat

$$u_2 - u_1 = c_{v,avg}(T_2 - T_1)$$

$$h_2 - h_1 = c_{p,avg}(T_2 - T_1)$$

$$s_2 - s_1 = c_{p,avg} \cdot \ln\left(\frac{T_2}{T_1}\right) - R \cdot \ln\left(\frac{P_2}{P_1}\right)$$

where

$$c_{p,avg} = c_p\left(@\frac{T_1 + T_2}{2}\right) \text{ and}$$

$$c_{v,avg} = c_v\left(@\frac{T_1 + T_2}{2}\right)$$

Using Ideal Gas Tables

u and h read from table

$$s_2 - s_1 = \phi_2 - \phi_1 - R \cdot \ln\left(\frac{P_2}{P_1}\right)$$

Always Work in Absolute Temperature for an Ideal Gas