# ME 201
## Thermodynamics

### Homework #8 Solution

1. Ten grams of water at 15°C and 100 kPa completely fills a balloon. The balloon is then heated on the stove top at constant pressure until the temperature reaches 125°C. Determine the boundary work in kJ involved in this process. What is the boundary power, if the process takes 700 seconds?

**Solution:**

Substance Type: Compressible(water)
Process: Constant Pressure

| State 1 | State 2 |
|---|---|
| $T_1$ = 15°C | $T_2$ = 125°C |
| $P_1$ = 100 kPa | $P_2 = P_1$ = **100 kPa** |
| $v_1$ = *0.001001 m³/kg* | $v_2$ = *1.8163 m³/kg* |
| $m_1$ = 0.01 kg | $m_1$ = 0.01 kg |
| phase: *sub.liq.* | phase: *sup.vap.* |

*Italicized* values are from steam tables. **Bold** values are calculated.

For a constant pressure process our boundary work is given by

$$w_{bnd} = P(v_2 - v_1)$$

and then

$$W_{bnd} = m \cdot w_{bnd} \text{ and } \dot{W}_{bnd} = \frac{W_{bnd}}{\Delta t}$$

So to perform this calculation we will need to determine the two specific volumes.

At state 1, we know the pressure and temperature so that the state is fixed. Going to the saturation pressure table we find at 100kPa

$$T_{sat} = 99.63°C$$

so we have subcooled liquid. But our pressure is too low to use the compressed liquid tables, so we must treat our liquid water as an incompressible substance. However, we know we will need to couple its property evaluation to the stream tables. Then we use the protocol laid out for using compressible substance tables for an incompressible substance and write

$$v_1 = v_f\,(\text{at } 15°C) = 0.001001\ m^3/kg$$

At state 2 we know the temperature and since the process is isobaric

$$P_2 = P_1 = 100 \text{ kPa}$$

Our state is fixed and at 100 kPa and we know that the boiling temperature is 99.63°C so we must have superheated vapor. Then we may go to the superheat table and find

$$v_2 = 1.8163 \text{ m}^3/\text{kg}$$

Then calculating the boundary work

$$w_{bnd} = 100(1.8163 - 0.001001) = 181.53 \text{ kJ/kg}$$

with

$$W_{bnd} = m \cdot w_{bnd} = (0.01)(181.53) = 1.82 \text{ kJ}$$

and

$$\dot{W}_{bnd} = \frac{1.82 \times 10^3}{700} = 2.59 \text{ W}$$

2. A piston-cylinder device of volume 0.05 $ft^3$ contains 0.001 $lb_m$ $H_2$ at 40 psia. The device undergoes a polytropic process with polytropic exponent 0.75 or

$$PV^{0.75} = \text{constant}$$

which increases the pressure to 110 psia.. Determine

(a) the boundary work in Btu for this process
(b) the final temperature

**Solution:**

Substance Type: Ideal Gas ($H_2$)
Process: Polytropic with n = 0.75

| State 1 | State 2 |
|---|---|
| $T_1$ = *375.8 R* | $T_2$ = *268.7 R* |
| $P_1$ = 40 psia | $P_2$ = 110 psia |
| $v_1$ = **50 $ft^3/lb_m$** | $v_2$ = **13 $ft^3/lb_m$** |
| $m_1$ = 0.001 $lb_m$ | $m_2$ = 0.001 $lb_m$ |
| $V_1$ = 0.05 $ft^3$ | $V_2$ = **0.013 $ft^3$** |

*Italicized* values are from ideal gas law. **Bold** values are calculated.
Note since we have a closed system our mass does not change.

For a polytropic process our boundary work is given by

$$W_{bnd} = \frac{P_2 V_2 - P_1 V_1}{1 - n}$$

At state 1 we know the pressure and can calculate the specific volume form the information given or

$$v_1 = \frac{V_1}{m_1} = \frac{0.05}{0.001} = 50\ \text{ft}^3/\text{lb}_m$$

We can then determine the temperature at state 1 from

$$T_1 = \frac{P_1 v_1}{R} = \frac{(40)(50)}{(10.73/2.016)} = 375.8\ \text{R}$$

At state 2 we know the pressure and have a polytropic process so that

$$P_2 V_2^n = P_1 V_1^n \text{ or solving}$$

$$V_2 = V_1\left(\frac{P_1}{P_2}\right)^{1/n} \text{ or substituting}$$

$$V_2 = (0.05)\left(\frac{40}{110}\right)^{1/0.75} = 0.013\ \text{ft}^3$$

We can now use our equation for the boundary work

$$W_{bnd} = \frac{P_2 V_2 - P_1 V_1}{1 - n} = \frac{(110)(0.013) - (40)(0.05)}{1 - 0.75}$$

$$= -2.29\ \text{psia} \cdot \text{ft}^3 \left(144\ \frac{\text{in}^2}{\text{ft}^2}\right)\left(\frac{1}{778}\ \frac{\text{Btu}}{\text{ft} \cdot \text{lb}_m}\right) = -0.424\ \text{Btu}$$

or a work **input** of 0.424 Btu. To obtain the final temperature we calculate the final specific volume from

$$v_2 = \frac{V_2}{m_2} = \frac{0.013}{0.001} = 13\ \text{ft}^3/\text{lb}_m$$

and then using the ideal gas law

$$T_2 = \frac{P_2 v_2}{R} = \frac{(110)(13)}{(10.73/2.016)} = 268.7\ \text{R}$$