ME 201
Thermodynamics

Homework #8 Solutions

1. A rigid wall, insulated container is divided into two regions by a removable wall. One region contains 0.1 lbm of CO at 100°F, while the other region contains 0.2 lbm of CO at 150°F. A stirrer is inserted into the container and when the wall between the two regions is removed the stirrer provides a shaft work input of 15 Btu. Determine the final temperature of the CO.

   Solution:

   System Type: Closed System
   Working Fluid: CO (Ideal Gas)
   Process: Constant Volume

<table>
<thead>
<tr>
<th>Initial State</th>
<th>Final State</th>
</tr>
</thead>
<tbody>
<tr>
<td>State 1</td>
<td>State 2</td>
</tr>
<tr>
<td>T₁ = 100°F = 310.93 K</td>
<td>T₂ = 150°F = 338.71 K</td>
</tr>
<tr>
<td>m₁ = 0.1 lbₘ = 0.0454 kg</td>
<td>m₂ = 0.2 lbₘ = 0.0908 kg</td>
</tr>
<tr>
<td>u₁ = 9.62 kJ/kg</td>
<td>u₂ = 30.23 kJ/kg</td>
</tr>
</tbody>
</table>

   Bold values are calculated.

   Q = 0
   Wₚ = -15 Btu (into system)
       = -15.82 kJ
   Wₚₙ = 0
   Initial State: Fixed
   Final State: UNKNOWN

   Conservation of Mass: m₁ + m₂ = m₃
   1st Law: m₃u₃ - (m₁u₁ + m₂u₂) = -Wₚ

   Approach: To fix the final state we will use our process description along with the conservation of mass to determine the final mass. Using the conservation of energy, we can determine the final internal energy. This will then fix the final state and we can go to the ideal gas tables to determine the final temperature.

   Using conservation of mass
   m₃ = m₁ + m₂ = 0.0454 + 0.0908 = 0.1362 kg

   Applying our conservation of energy we can write
   m₃u₃ - m₁u₁ - m₂u₂ = -Wₚ

   From the ideal gas tables for CO we find
   u₁ = 9.62 kJ/kg and u₂ = 30.23 kJ/kg

   Using this in our conservation of energy equation and solving for u₃
   u₃ = \( \frac{(m₁u₁ + m₂u₂) - Wₚ}{m₃} \)

   Substituting our values and calculating we have
   u₃ = 139.59 kJ/kg
Going to the CO table, we find that
\[ T_3 = 484.3 \, \text{K} = 412.1 \, ^\circ\text{F} \]

2. A balloon containing 0.5 kg of Refrigerant-12 is removed from the freezer at 100 kPa and 
-40\(^\circ\)C and placed on the kitchen counter. Determine the heat transfer required to

(a) raise the R-12 temperature to its boiling point
(b) boil the liquid
(c) raise the resulting vapor to the room temperature of 23\(^\circ\)C

You may assume that the kitchen pressure is 100 kPa.

**Solution:**
For this problem we will consider the kitchen as our system.

**System Type:** Closed System

**Working Fluid:** R-12 (Phase Change substance)

**Process:** Constant pressure

<table>
<thead>
<tr>
<th>State 1</th>
<th>State 2</th>
<th>State 3</th>
<th>State 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>T(_1) = -40(^\circ)C</td>
<td>T(_2) = -29.87(^\circ)C</td>
<td>T(_3) = -29.87(^\circ)C</td>
<td>T(_4) = 23(^\circ)C</td>
</tr>
<tr>
<td>P(_1) = 100 kPa</td>
<td>P(_2) = 100 kPa</td>
<td>P(_3) = 100 kPa</td>
<td>P(_4) = 100 kPa</td>
</tr>
<tr>
<td>v(_1) = 0.000659 m(^3)/kg</td>
<td>v(_2) = 0.000672 m(^3)/kg</td>
<td>v(_3) = 0.158607 m(^3)/kg</td>
<td>v(_4) = 0.1994 m(^3)/kg</td>
</tr>
<tr>
<td>u(_1) = -0.07 kJ/kg</td>
<td>u(_2) = 8.88 kJ/kg</td>
<td>u(_3) = 158.39 kJ/kg</td>
<td>u(_4) = 185.75 kJ/kg</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Process 1-2</th>
<th>Process 2-3</th>
<th>Process 3-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q(_{1-2}) = UNKNOWN</td>
<td>Q(_{2-3}) = UNKNOWN</td>
<td>Q(_{3-4}) = UNKNOWN</td>
</tr>
<tr>
<td>W(_{sh}) = 0</td>
<td>W(_{sh}) = 0</td>
<td>W(_{sh}) = 0</td>
</tr>
<tr>
<td>W(_{bnd,1-2}) = UNKNOWN</td>
<td>W(_{bnd,2-3}) = UNKNOWN</td>
<td>W(_{bnd,3-4}) = UNKNOWN</td>
</tr>
<tr>
<td>Initial State: Fixed</td>
<td>Initial State: Fixed</td>
<td>Initial State: Fixed</td>
</tr>
<tr>
<td>Final State: Fixed</td>
<td>Final State: Fixed</td>
<td>Final State: Fixed</td>
</tr>
</tbody>
</table>

*Italicized* values are from steam tables.

Conservation of Mass
\[ m_1 = m_2 = m_3 = m_4 \]

1st Law
\[ u_2 - u_1 = q_{1-2} - w_{bnd,1-2} \]
\[ u_3 - u_2 = q_{2-3} - w_{bnd,2-3} \]
\[ u_4 - u_3 = q_{3-4} - w_{bnd,3-4} \]

**W\(_{bnd}\)**
\[ W_{bnd,1-2} = P_1(v_2 - v_1) \]
\[ W_{bnd,2-3} = P_2(v_3 - v_2) \]
\[ W_{bnd,3-4} = P_3(v_4 - v_3) \]

Approach: The three heat transfers will be calculated from the first law, but to do this we must
first evaluate the boundary work using the appropriate expression for \(\int Pdv\). Since all of our states
are fixed it will simply be a matter of evaluating our needed properties. We begin by noting that
all four states can be treated as phase change substances, so that if we desire to use the R-12
tables we can write

\[ v_1 = v_f(\text{at } -40\(^\circ\)C) = 0.000659 \, m^3/kg \]
\[ u_1 = u_f(\text{at } -40\(^\circ\)C) = -0.07 \, kJ/kg \]
\[ v_2 = v_f(\text{at 100 kPa}) = 0.000672 \, m^3/kg \]
\[ u_2 = u_f(\text{at 100 kPa}) = 8.88 \, kJ/kg \]
\[ v_3 = v_g(\text{at 100 kPa}) = 0.158607 \, m^3/kg \]
\[ u_3 = u_g(\text{at 100 kPa}) = 158.39 \, kJ/kg \]
\[ v_4 = v(\text{at 23\(^\circ\)C, 100 kPa}) = 0.1994 \, m^3/kg \]
\[ u_4 = u(\text{at 23\(^\circ\)C, 100 kPa}) = 185.75 \, kJ/kg \]
Now calculating our boundary work

\[ w_{bnd,1-2} = P_1(v_2 - v_1) = (100)(0.000672 - 0.000659) = 1.3000 \times 10^{-3} \text{kJ/kg} \]
\[ w_{bnd,2-3} = P_2(v_3 - v_2) = (100)(0.158607 - 0.000672) = 1.5794 \times 10^1 \text{kJ/kg} \]
\[ w_{bnd,3-4} = P_3(v_4 - v_3) = (100)(0.1994 - 0.158607) = 4.0793 \text{kJ/kg} \]

and our heat transfer is

\[ q_{1-2} = u_2 - u_1 + w_{bnd,1-2} = 8.88 - (-0.07) + (1.3000 \times 10^{-3}) = 8.95 \text{kJ/kg} \]
\[ q_{2-3} = u_3 - u_2 + w_{bnd,2-3} = 158.39 - 8.88 + (1.5794 \times 10^1) = 165.30 \text{kJ/kg} \]
\[ q_{3-4} = u_4 - u_3 + w_{bnd,3-4} = 185.75 - 158.39 + (4.0793) = 31.44 \text{kJ/kg} \]

or on a total basis

\[ Q_{1-2} = (0.5)(8.95) = 4.48 \text{kJ} \]
\[ Q_{2-3} = (0.5)(165.30) = 82.65 \text{kJ} \]
\[ Q_{3-4} = (0.5)(31.44) = 15.72 \text{kJ} \]

3. Three of the processes that occur in the piston cylinder device of an internal combustion engine are:

Process 1: Constant pressure heat addition during which the volume doubles
Process 2: Isentropic expansion during which the volume increases 9 times
Process 3: Constant volume heat removal to a final pressure of 100 kPa

At the start of process 1 the engine contains 0.035 liters of air at 950 K and 5.7 MPa. For each of the three processes the air undergoes find the work and heat transfer in kJ. What is the total work and heat transfer for all three processes together?

**Solution:**

We will work this problem process by process.

<table>
<thead>
<tr>
<th>State 1</th>
<th>State 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>T₁ = 950 K</td>
<td>T₂ = 1900 K</td>
</tr>
<tr>
<td>P₁ = 5700 kPa</td>
<td>P₂ = 5700 kPa</td>
</tr>
<tr>
<td>V₁ = 0.035 liters = 3.5 \times 10^{-5} \text{m}³</td>
<td>V₂ = 7.0 \times 10^{-5} \text{m}³</td>
</tr>
<tr>
<td>m₁ = 7.317 \times 10^{-4} \text{kg}</td>
<td>m₂ = 7.317 \times 10^{-4} \text{kg}</td>
</tr>
<tr>
<td>u₁ = 489.85 kJ/kg</td>
<td>u₂ = 1374.70 kJ/kg</td>
</tr>
<tr>
<td>v₁ = 4.783 \times 10^{-2} \text{m}³/kg</td>
<td>v₂ = 9.566 \times 10^{-2} \text{m}³/kg</td>
</tr>
</tbody>
</table>

*Italicized* values are from ideal gas relations or air tables. *Bold* values are calculated.

\[ Q = \text{UNKNOWN} \]
\[ W_{sh} = 0 \]
\[ W_{bnd} = P(V₂ - V₁) \text{ (isobaric)} \]
Initial State: Fixed
Final State: UNKNOWN
Conservation of Mass: m₁ = m₂
1st Law: U₂ - U₁ = Q - W_{bnd}
Approach: To fix the final state we will use our process description along with the conservation of mass to determine the final specific volume. The boundary work is then calculated. Finally we use the conservation of energy to determine the heat transfer.

We start by determining our mass. Using the ideal gas law we have

\[ v_1 = \frac{RT_1}{P_1} = \frac{(0.287)(950)}{5700} = 4.783 \times 10^{-2} \text{ m}^3/\text{kg} \]

\[ m_1 = \frac{V_1}{v_1} = \frac{(3.5 \times 10^{-5})}{(4.783 \times 10^{-2})} = 7.317 \times 10^{-4} \text{ kg} \]

From the air tables we find

\[ u_1 = 489.85 \text{ kJ/kg} \]

By conservation of mass we have

\[ m_2 = m_1 = 7.317 \times 10^{-4} \text{ kg} \]

Since our process is isobaric

\[ P_2 = P_1 = 5700 \text{ kPa} \]

We are told that the volume doubles, so that

\[ V_2 = 2V_1 = 7.0 \times 10^{-5} \text{ m}^3 \]

\[ v_2 = 2v_1 = 9.566 \times 10^{-2} \text{ m}^3/\text{kg} \]

The temperature at state 2 is then calculated from

\[ T_2 = \frac{P_2v_2}{R} = \frac{(5700)(9.566 \times 10^{-2})}{0.287} = 1900 \text{ K} \]

Using the air tables:

\[ u_2 = 1374.70 \text{ kJ/kg} \]

Our boundary work is calculated from

\[ W_{\text{bnd}} = P(V_2-V_1) = (5700)(7.35)(10^{-5}) = 0.1995 \text{ kJ} \]

The first law is then used to calculate the heat transfer or

\[ Q = m(u_2 - u_1) + W_{\text{bnd}} = (7.317 \times 10^{-4})(1374.70 - 489.85) + 0.1995 = 0.8467 \text{ kJ} \]

For our second process we have

| System Type: Closed System |
| Working Fluid: Air (ideal gas) |
| Process: Isentropic |

<table>
<thead>
<tr>
<th>State 2</th>
<th>State 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>T_2 = 1900 K</td>
<td>T_3 = 948.1 K</td>
</tr>
<tr>
<td>P_2 = 5700 kPa</td>
<td>P_3 = 316.1 kPa</td>
</tr>
<tr>
<td>V_2 = 7 \times 10^{-3} \text{ m}^3</td>
<td>V_3 = 6.3 \times 10^{-4} \text{ m}^3</td>
</tr>
<tr>
<td>m_2 = 7.317 \times 10^{-4} \text{ kg}</td>
<td>m_3 = 7.317 \times 10^{-4} \text{ kg}</td>
</tr>
<tr>
<td>u_2 = 1374.70 kJ/kg</td>
<td>u_3 = 505.16 kJ/kg</td>
</tr>
<tr>
<td>V_2 = 9.566 \times 10^{-2} \text{ m}^3/\text{kg}</td>
<td>V_3 = 0.8609 \text{ m}^3/\text{kg}</td>
</tr>
<tr>
<td>v_{r2} = 0.005099</td>
<td>v_{r3} = 0.0459</td>
</tr>
</tbody>
</table>

*Italicized* values are from ideal gas relations. **Bold** values are calculated.

\[ Q = \text{UNKNOWN} \]

\[ W_{\text{sh}} = 0 \]
\[ W_{\text{bnd}} = \frac{P_3 V_3 - P_2 V_2}{1 - k} \quad \text{(isentropic)} \]

Initial State: Fixed
Final State: UNKNOWN
Conservation of Mass: \( m_2 = m_3 \)
1st Law: \( U_3 - U_2 = Q - W_{\text{bnd}} \)

Approach: For the final state we will apply our process description to get the final volumes, then the isentropic condition to determine the temperature and then calculate our pressure from the ideal gas law. Then our boundary work will be calculated from \( \int P \, dV \). Finally, we use the conservation of energy to determine the heat transfer.

We begin by applying conservation of mass to write
\[
m_3 = m_2 = 7.317 \times 10^{-4} \text{ kg}
\]

We are told that our volume increases nine fold, so that
\[
V_3 = 9V_2 = 6.3 \times 10^{-4} \text{ m}^3
\]
\[
v_3 = 9v_2 = 8.609 \times 10^{-1} \text{ m}^3/\text{kg}
\]

To determine the temperature at 3 we find that we can use the \( v_r \) in the air tables as follows. For an isentropic process we have
\[
\frac{v_{r3}}{v_{r2}} = \left( \frac{v_3}{v_2} \right) = (0.005099)(9) = 0.0459
\]

where \( v_{r2} \) is obtained from the air tables at 1900 K. Now going to the air tables for a \( v_r \) of 0.0459 we find
\[
T_3 = 948.1 \text{ K and } u_3 = 505.16 \text{ kJ/kg}
\]

We then calculate the pressure by
\[
P_3 = \frac{RT_3}{v_3} = \frac{(0.287)(948.1)}{(0.8609)} = 316.1 \text{ kPa}
\]

The boundary work is calculated from
\[
W_{\text{bnd}} = \frac{P_3 V_3 - P_2 V_2}{1 - k} = \frac{(316.1)(6.3 \times 10^{-4}) - (5700)(7 \times 10^{-5})}{1-1.3126} = 0.6393 \text{ kJ}
\]

where \( k \) has been taken at 1424 K. Now using our conservation of energy to determine the heat transfer
\[
Q = m(u_3 - u_2) + W_{\text{bnd}}
\]
\[
= (7.317 \times 10^{-4})(505.16 - 1374.70) + 0.6393
\]
\[
= 0.0031 \text{ kJ}
\]
Finally, we have our third process

System Type: Closed System
Working Fluid: Air (ideal gas)
Process: Isotropic

<table>
<thead>
<tr>
<th>State 3</th>
<th>State 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_3 = 948.1$ K</td>
<td>$T_4 = 300$ K</td>
</tr>
<tr>
<td>$P_3 = 316.1$ kPa</td>
<td>$P_4 = 100$ kPa</td>
</tr>
<tr>
<td>$V_3 = 6.3 \times 10^{-4}$ m$^3$</td>
<td>$V_4 = 6.3 \times 10^{-4}$ m$^3$</td>
</tr>
<tr>
<td>$m_3 = 7.317 \times 10^{-4}$ kg</td>
<td>$m_4 = 6.2718 \times 10^{-5}$ kg</td>
</tr>
<tr>
<td>$u_3 = 505.16$ kJ/kg</td>
<td>$u_4 = 1.44$ kJ/kg</td>
</tr>
<tr>
<td>$v_3 = 0.8609$ m$^3$/kg</td>
<td>$v_4 = 0.8609$ m$^3$/kg</td>
</tr>
</tbody>
</table>

*Italicized* values are from ideal gas relations or tables. *Bold* values are calculated.

$Q = \text{UNKNOWN}$

$W_{sh} = 0$

$W_{bnd} = 0$ (constant volume process)

Initial State: Fixed

Final State: UNKNOWN

Conservation of Mass: $m_4 = m_3$

1st Law: $U_4 - U_3 = Q$

Approach: To fix the final state we will use our process description. The ideal gas law can then be used to determine the temperature and the internal energy is found from the air tables. Finally, we use the conservation of energy to determine the heat transfer.

Our conservation of mass yields

$m_4 = m_3 = 7.317 \times 10^{-4}$ kg

Our process is isometric so that

$V_4 = V_3 = 6.3 \times 10^{-4}$ m$^3$

$v_4 = v_3 = 0.8609$ m$^3$/kg

The temperature is determined from

$$T_4 = \frac{P_4 v_4}{R} = \frac{(100)(0.8609)}{(0.287)} = 300$ K

Then from the air tables at 300 K

$u_4 = 1.44$ kJ/kg

The first law is then used to calculate the heat transfer or

$$Q = m(u_4 - u_3) = (7.317 \times 10^{-4})(1.44 - 505.16)$$

$$= -0.3686$ kJ

Then the total heat transfer and boundary work are

$Q = 0.4812$ kJ and $W_{bnd} = 0.8388$ kJ