ME 201
Thermodynamics

Homework #4 Solutions

1. For the two processes given below, determine the final temperature, pressure, specific volume, and the changes in internal energy, enthalpy, and entropy.

   a. Air at 4500 K and 450 kPa goes isothermally to 8 kPa.
   Solution:

      Substance Type: Ideal Gas (Air)
      Problem Type: Process (Isothermal)

      | State 1 | State 2 |
      |---------|---------|
      | T₁ = 4500 K | T₂ = 4500 K |
      | P₁ = 450 kPa | P₂ = 8 kPa |

      We note that to fix state 2 we need another property, but since the process is isothermal we must have
      \[ T₂ = T₁ = 4500 \text{ K} \]

      Then the final specific volume can be calculated from the ideal gas law or
      \[ v₂ = \frac{RT₂}{P₂} = \frac{(0.286)(4500)}{8} = 161.43 \text{ m}^3/\text{kg} \]

      To calculate the property changes we would like to use our tables. Unfortunately, our tables do not go up this high. However, we know that since air is an ideal gas and the process is isothermal (no temperature change) we must have
      \[ \Delta u = 0 \text{ kJ/kg} \text{ and } \Delta h = 0 \text{ kJ/kg} \text{ and } \phi₁ = \phi₂ \]

      For the entropy change, we will have
      \[ \Delta s = - R \cdot \ln \left( \frac{P₂}{P₁} \right) \]
      \[ = - (0.286) \ln \left( \frac{8}{450} \right) \]
      \[ = 1.1566 \text{ kJ/(kg \cdot K)} \]

   b. Hydrogen at 2033 K and 132 kPa goes isentropically to 497 K.
   Solution:

      Substance Type: Ideal Gas (Hydrogen)
      Problem Type: Process (Isentropic)

      | State 1 | State 2 |
      |---------|---------|
      | T₁ = 2033 K | T₂ = 497 K |
      | P₁ = 132 kPa | P₂ = 0.713 kPa |

      Bold values are calculated
We note that to fix state 2 we need another property, but since the process is isentropic, we will have
\[ \Delta s = 0 = \phi_2 - \phi_1 - R \cdot \ln \left( \frac{P_2}{P_1} \right) \]

Since we know our temperature at 2, and we can find \( \phi_2 \) from the gas tables, we can use the above equation to calculate the pressure or
\[ P_2 = P_1 \exp \left( \frac{\phi_2 - \phi_1}{R} \right) \]

Going to our ideal gas tables for H\(_2\) (Table A.6SI), we find
\[
\begin{align*}
  h_1 &= 26829.75 \text{ kJ/kg} & u_1 &= 19674.6 \text{ kJ/kg} & \phi_1 &= 93.7419 \text{ kJ/(kg} \cdot \text{K}) \\
  h_2 &= 2874.47 \text{ kJ/kg} & u_2 &= 2053.80 \text{ kJ/kg} & \phi_2 &= 72.2066 \text{ kJ/(kg} \cdot \text{K})
\end{align*}
\]

Then our changes are
\[
\begin{align*}
  \Delta h &= h_2 - h_1 = 2874.47 - 26829.75 = -23955.28 \text{ kJ/kg} \\
  \Delta u &= u_2 - u_1 = 2053.80 - 19674.6 = -17620.8 \text{ kJ/kg} \\
  \Delta s &= 0 \text{ kJ/(kg} \cdot \text{K})
\end{align*}
\]

The final pressure is given by
\[ P_2 = P_1 \exp \left( \frac{\phi_2 - \phi_1}{R} \right) = (132) \exp \left( \frac{72.2066 - 93.7419}{4.1243} \right) = 0.713 \text{ kPa} \]

Now solving for the specific volume from the ideal gas law
\[ v_2 = \frac{RT_2}{P_2} = \frac{(4.1243)(487)}{0.713} = 2818.396 \text{ m}^3/\text{kg} \]

2. Calculate the final temperature and change in specific internal energy as steel at 212 kPa and 533 K goes isenthalpically to 640 kPa.

**Solution:**

Substance Type: Incompressible (steel)

Problem Type: Process (isenthalpic, constant \( h \))

<table>
<thead>
<tr>
<th>State 1</th>
<th>State 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_1 ) = 533 K</td>
<td>( T_2 = 499.76^\circ \text{F} )</td>
</tr>
<tr>
<td>( P_1 = 212 \text{ kPa} )</td>
<td>( P_2 = 640 \text{ kPa} )</td>
</tr>
</tbody>
</table>

**Bold** values are calculated

We note that to fix state 2 we need another property, but since the process is isenthalpic, we will have
\[ \Delta h = 0 = c_{P,avg} (T_2 - T_1) + v_{avg} (P_2 - P_1) \]

or solving for \( T_2 \)
\[ T_2 = T_1 - \frac{v_{avg} (P_2 - P_1)}{c_{P,avg}} \]

Rigorously, we would like to evaluate \( v_{avg} \) and \( c_{P,avg} \) at \((T_1+T_2)/2\), but we do not know \( T_2 \). As a first approximation, we will evaluate them at \( T_1 \) and then iterate if needed. At 533 K we find from Table B.9SI
\[ c_p = 537.77 \text{ J/(kg} \cdot \text{K}) \quad \rho = 7821.2 \text{ kg/m}^3 \]
Then

\[ v_{\text{avg}} = \frac{1}{\rho} = \frac{1}{7821.2} = 1.2786 \times 10^{-4} \text{ m}^3/\text{kg} \]

and

\[ T_2 = 533 - \frac{(1.2786 \times 10^{-4})(90 - 30)(6.895 \text{ kPa/psia})(1000 \text{ Pa/kPa})}{(537.77)} = 532.9 \text{ K} \]

Then the change in internal energy is

\[ \Delta u = c_{p,\text{avg}} (T_2 - T_1) = (537.77)(-0.1) = -53.78 \text{ J/kg} \]

3. Determine the changes in internal energy, enthalpy, and entropy as engine oil at 70 kPa and 298 K goes to 100 kPa and 420 K.

\textbf{Solution:}

\begin{align*}
\text{Substance Type: Incompressible (Liquid Petroleum)} \\
\text{Problem Type: Process} \\
\begin{array}{ll}
\text{State 1} & \text{State 2} \\
T_1 = 298 \text{ K} & T_2 = 420 \text{ K} \\
P_1 = 70 \text{ kPa} & P_2 = 100 \text{ kPa}
\end{array}
\end{align*}

For engine oil, we can use Table B.2SI to obtain our \( c_p \) and density. Calculating our average temperature, we have

\[ T_{\text{avg}} = \frac{T_1 + T_2}{2} = \frac{298 + 420}{2} = 359 \text{ K} \]

Then from the table

\[ c_{p,\text{avg}} = 2.6772 \text{ kJ/(kg \cdot K)} \]

\[ \rho = 1072.04 \text{ kg/m}^3 \]

\[ v_{\text{avg}} = \frac{1}{\rho} = \frac{1}{1072.04} = 9.328 \times 10^{-4} \text{ m}^3/\text{kg} \]

which gives

\[ \Delta u = c_{p,\text{avg}}(T_2 - T_1) = (2.6772)(420 - 298) = 326.62 \text{ kJ/kg} \]

\[ \Delta h = c_{p,\text{avg}}(T_2 - T_1) + v_{\text{avg}}(P_2 - P_1) \\
= (2.6772)(420 - 298) + (9.328 \times 10^{-4})(100 - 70) = 326.65 \text{ kJ/kg} \]

\[ \Delta s = c_{p,\text{avg}} \ln \left( \frac{T_2}{T_1} \right) = (2.6772)\ln \left( \frac{420}{298} \right) = 0.9187 \text{ kJ/kg \cdot K} \]
4. What are the enthalpy, internal energy, specific volume, and entropy for steam at 1107°C and 27 MPa?

Solution:

Substance Type: Phase Change (steam)
Problem Type: State

We are given steam at 27 MPa and 1107°C. First, we must determine the fluid phase. We go to the saturation pressure table, C.2bSI, and find the boiling temperature at 27 MPa. However, we note that the table does not go up that far. This means we are beyond the critical point and can treat the steam as superheated vapor.

So now we go to the superheat table, C.1cSI, and find that again we do not have a table entry at either 27 MPa or 1107°C. This means we will have to set up a double interpolation. To do so we set up the table shown below and enter the values from the steam tables at 25 MPa, 30 MPa, and 1100°C, 1200°C.

<table>
<thead>
<tr>
<th>T</th>
<th>P=25 MPa</th>
<th>P=27 MPa</th>
<th>P=30 MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>v</td>
<td>h</td>
<td>u</td>
</tr>
<tr>
<td>1100</td>
<td>0.0251</td>
<td>4814.4</td>
<td>4186.5</td>
</tr>
<tr>
<td>1107</td>
<td>0.0261</td>
<td>49482</td>
<td>4295</td>
</tr>
<tr>
<td>1200</td>
<td>0.0271</td>
<td>5103.5</td>
<td>7.7710</td>
</tr>
</tbody>
</table>

We begin our double interpolation by interpolating in pressure, so, for example, at 1100°C for specific volume we have

\[
\frac{v(27 \text{ MPa}) - v(25 \text{ MPa})}{v(30 \text{ MPa}) - v(25 \text{ MPa})} = \frac{27 \text{ MPa} - 25 \text{ MPa}}{30 \text{ MPa} - 25 \text{ MPa}}
\]

or solving for \(v(27 \text{ MPa})\)

\[
v(27 \text{ MPa}) = 0.0251 + (0.0209 - 0.0251) \frac{27 - 25}{30 - 25}
\]

\[= 0.0234 \text{ m}^3/\text{kg}
\]

Carrying out these calculations for \(v, u, h,\) and \(s\) at \(P=27 \text{ MPa}\) and both 1100°C and 1150°C and entering in our table gives

<table>
<thead>
<tr>
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<th>P=30 MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>v</td>
<td>h</td>
<td>u</td>
</tr>
<tr>
<td>1100</td>
<td>0.02517</td>
<td>4206.1</td>
<td>4835.4</td>
</tr>
<tr>
<td>1107</td>
<td>0.02716</td>
<td>4424.6</td>
<td>5103.5</td>
</tr>
<tr>
<td>1200</td>
<td>0.02763</td>
<td>4539.5</td>
<td>5324.5</td>
</tr>
</tbody>
</table>

Now we do the temperature interpolation to give us the properties at 27 MPa and 1107°C.

<table>
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<tr>
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<td>0.0251</td>
<td>4814</td>
<td>4187</td>
</tr>
<tr>
<td>1107</td>
<td>0.0261</td>
<td>4948</td>
<td>4295</td>
</tr>
<tr>
<td>1200</td>
<td>0.0266</td>
<td>5088</td>
<td>4425</td>
</tr>
</tbody>
</table>