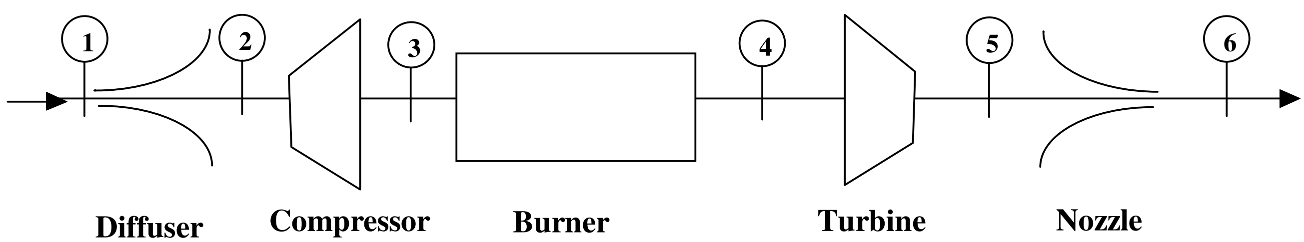
ME 201
Thermodynamics

Homework #21 Solution

1. Consider a jet aircraft flying at 300 m/s at an altitude of 3,000 m (use Table A-16 in the text to determine the pressure and temperature). The jet operates with a simple, ideal turbojet engine. The burner of the engine operates at 1300 kPa and 1800 K. Determine

    a. specific thrust produced
    b. propulsive efficiency

Solution:
We begin with a block diagram of the system.



Now we set up our table.

Node	T(K)	P(kPa)	h(kJ/kg)	ϕ(kJ/kg·K)	$\vec{v}$ (m/s)
1	268.5	70.12	*268.62*	*1.5907*	300
2	*313.4*	**120.4**	**313.4**	*1.7458*	0
3	*611.3*	1300	*618.93*	**2.4287**	0
4	1800	1300	*2003.3*	*3.6684*	0
5	*1548.6*	**683.4**	**1697.77**	*3.4838*	0
6	*885.1*	70.12	*916.23*	**2.829812**	**1250**

Italicized values form air tables
Bold values are calculated

We begin by entering our operating information. Since state 1 and state 4 are fixed from the operating information we can go to the air tables and find

$h_1 = 268.62$ kJ/kg and $\phi_1 = 1.5907$ kJ/(kg·K)

$h_4 = 2003.3$ kJ/kg and $\phi_4 = 3.6684$ kJ/(kg·K)

We now traverse the cycle.

1-2 Diffuser: Isentropic

The 1st law gives

$$h_2 = h_1 + \frac{\vec{v}_1^2}{2} = 268.62 + \frac{(300)^2}{2}(10^{-3}) = 313.62 \text{ kJ/kg}$$

We can then go to the air tables and find

$T_2 = 313.4$ K and $\phi_2 = 1.7458$ kJ/(kg·K)

The pressure can be determine from the isentropic condition

$$0 = \phi_2 - \phi_1 - R \cdot \ln\left(\frac{P_2}{P_1}\right)$$

or

$$P_2 = P_1 \exp\left\{\frac{\varphi_2 - \varphi_1}{R}\right\} = (70.12)\exp\left\{\frac{1.7458 - 1.5907}{0.287}\right\} = 120.4 \text{ kPa}$$

2-3 Compressor: Isentropic

Our isentropic condition is

$$0 = \phi_3 - \phi_2 - R \cdot \ln\left(\frac{P_3}{P_2}\right)$$

or

$$\varphi_3 = \varphi_2 + R \cdot \ln\left(\frac{P_3}{P_2}\right) = 1.7458 + (0.287)\ln\left(\frac{1300}{120.4}\right) = 2.4287 \text{ kJ/(kg} \cdot \text{K)}$$

From the air tables we find

$T_3 = 611.3$ K and h_3 =618.93 kJ/kg

3-4 Burner: Isobaric

All data is already known.

4-5 Turbine: Isentropic

The 1st law gives

$w_t = h_4 - h_5 = -w_c = h_3 - h_2$

or

$h_5 = h_4 - h_3 + h_2 = 2003.3 - 628.93 + 313.62 = 1697.77$ kJ/kg

We can then go to the air tables and find

$T_5 = 1548.6$ K and $\phi_5 = 3.4838$ kJ/(kg·K)

The pressure can be determine from the isentropic condition

$$0 = \phi_5 - \phi_4 - R \cdot \ln\left(\frac{P_5}{P_4}\right)$$

or

$$P_5 = P_4 \exp\left\{\frac{\phi_5 - \phi_4}{R}\right\} = (1300)\exp\left\{\frac{3.4838 - 3.6684}{0.287}\right\} = 683.4 \text{ kPa}$$

<u>5-6 Nozzle: Isentropic</u>

Our isentropic condition is

$$0 = \phi_6 - \phi_5 - R \cdot \ln\left(\frac{P_6}{P_5}\right)$$

or

$$\phi_6 = \phi_5 + R \cdot \ln\left(\frac{P_6}{P_5}\right) = 3.4838 + (0.287)\ln\left(\frac{70.12}{683.4}\right) = 2.8298 \text{ kJ/(kg} \cdot \text{K)}$$

From the air tables we find

$T_6 = 885.1$ K and $h_6 = 916.23$ kJ/kg

The velocity is then calculated from the 1st law

$$h_5 = h_6 + \frac{\vec{v}_6^2}{2}$$

or

$$\vec{v}_6 = \sqrt{2(h_5 - h_6)} = \sqrt{2(1697.77 - 916.23)(10^3)} = 1250 \text{ m/s}$$

<u>System Calculations</u>

Specific thrust

$$f_t = \vec{v}_6 - \vec{v}_1 = 1250 - 300 = 950 \text{ m/s} = 950 \text{ N} \cdot \text{s/kg}$$

Propulsive efficiency

$$\eta_p = \frac{\dot{W}_p}{\dot{Q}_{in}}$$

Propulsive power

$$\dot{W}_p = \dot{m}(\vec{v}_6 - \vec{v}_1)\vec{v}_{aircraft} = (285.1)\dot{m} \text{ kW}$$

Heat transfer rate in

$$\dot{Q}_{in} = \dot{m}(h_4 - h_3) = (1384.37)\dot{m} \text{ kW}$$

which gives a propulsive efficiency of 0.206 or 20.6%.