ME 201
Thermodynamics

Homework #20 Solution

1. Consider an internal combustion engine operating on the ideal Dual cycle with the following conditions:

   Two cylinder, four stroke engine with displacement of 1.6 liters
   Compression ratio of 7.5
   Cutoff Ratio of 1.7
   Combustion temperature of 1700 K
   Engine speed of 1300 rpm

   The initial temperature and pressure are taken to be 150 kPa and 315 K due to turbocharging of the intake air. Just before the final process (constant volume cooling) 85% of the air is extracted and is used to power a turbine that supplies power to the intake compressor (i.e. the turbocharger). Determine

   a. engine thermal efficiency
   b. engine power output
   c. engine MEP
   d. exhaust turbine power

Solution:
We begin by making the calculations for our V_{BDC} and V_{TDC}. We first calculate our displacement volume per cylinder

$$\left(V_{disp}\right)_{cyl} = \frac{\left(V_{disp}\right)_{engine}}{N_{cyl}} = \frac{1.6 \text{ x } 10^{-3}}{2} = 8 \text{ x } 10^{-4} \text{ m}^3$$

From class we found that

$$V_{TDC} = \frac{\left(V_{disp}\right)_{cyl}}{r - 1} = \frac{8 \text{ x } 10^{-4}}{7.5 - 1} = 1.23 \text{ x } 10^{-4} \text{ m}^3$$

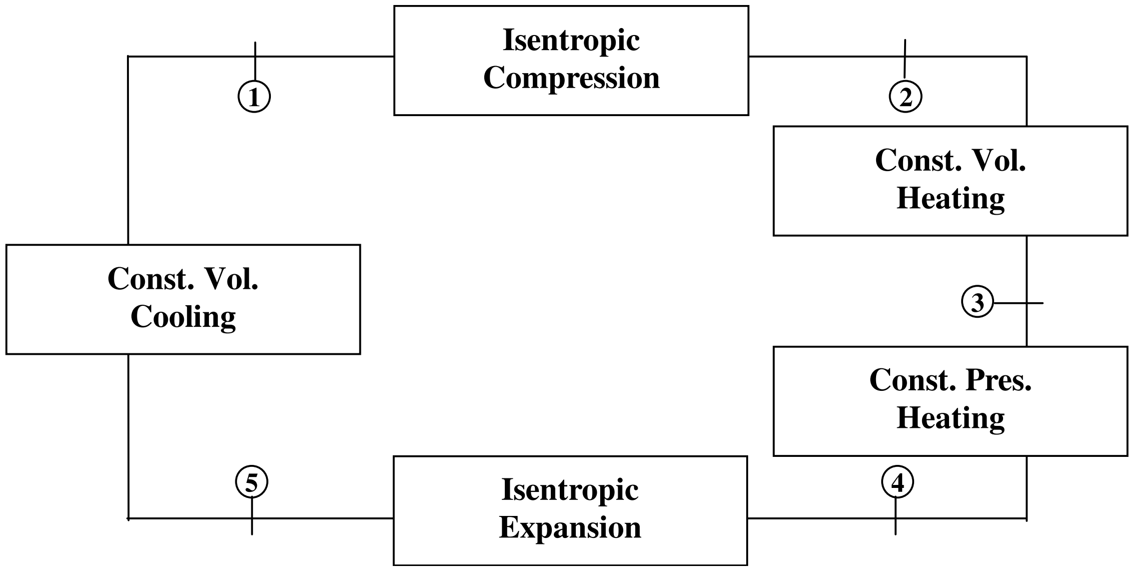
and

$$V_{BDC} = \left(V_{disp}\right)_{cyl} + V_{TDC} = 8 \text{ x } 10^{-4} + 1.23 \text{ x } 10^{-4} = 9.23 \text{ x } 10^{-4} \text{ m}^3$$

Our volume following the constant pressure heating, V_4, is given by the cutoff ratio

$$V_4 = r_c V_{TDC} = (1.7)(1.23 \text{ x } 10^{-4}) = 2.09 \text{ x } 10^{-4} \text{ m}^3$$

We now layout our block diagram for the cycle.



Next we set up our node table.

Node	T(K)	P(kPa)	v(m³/kg)	V(m³)	u(kJ/kg)	φ(kJ/kg K)
1	315	150	**0.6027**	9.23×10^{-4}	*224.85*	*1.7511*
2	*687*	*2453*	**0.0803**	1.23×10^{-4}	*502.57*	*2.5536*
3	*1001*	**3572**	**0.0803**	1.23×10^{-4}	*759.42*	*2.9683*
4	1700	*3572*	**0.1365**	2.09×10^{-4}	*1392.7*	*3.5979*
5	*1062*	*505*	**0.6027**	9.23×10^{-4}	*812.32*	*3.0366*

Italicized values are from air tables or ideal gas law
Bold values are calculated

We enter our operating information on the tables. Next we calculate the mass in the cylinder with

$$m = \frac{P_1 V_1}{R T_1} = \frac{(150)(9.23 \times 10^{-4})}{(0.287)(315)} = 1.53 \times 10^{-3} \text{ kg}$$

This allows us to calculate the specific volumes at all our states using

$$v = \frac{V}{m}$$

For state 1 we have two properties, so the state is fixed and we can go to the air tables and find

$u_1 = 224.85$ kJ/kg and $\phi_1 = 1.7511$ kJ/(kg·K)

State 4 is also fixed so from the air tables

$u_4 = 1392.7$ kJ/kg and $\phi_4 = 3.5979$ kJ/(kg·K) and from the ideal gas equation

$$P_4 = \frac{mRT_4}{V_4} = \frac{(1.53 \text{ x } 10^{-3})(0.287)(1700)}{(2.09 \text{ x } 10^{-4})} = 3572 \text{ kPa}$$

Now we can traverse the cycle.

1-2 Isentropic compression

As we saw in class, it is much easier to use the v_r from the air tables for these isentropic processes, hence

$$\frac{v_1}{v_2} = \frac{V_1}{V_2} = \frac{v_{r1}}{v_{r2}}$$

so that

$$v_{r2} = v_{r1}\left(\frac{V_2}{V_1}\right) = (549.8)\left(\frac{1.23 \text{ x } 10^{-4}}{9.23 \text{ x } 10^{-4}}\right) = 73.27$$

Interpolating from the air tables we get

$T_2 = 687$ K and $u_2 = 502.57$ kJ/kg and $\phi_2 = 2.5536$ kJ/(kg·K)

and from the ideal gas equation

$$P_2 = \frac{mRT_2}{V_2} = \frac{(1.53 \text{ x } 10^{-3})(0.287)(687)}{(1.23 \text{ x } 10^{-4})} = 2453 \text{ kPa}$$

2-3 Constant Volume Heating

There is no additional information provided from this process that will help us fix the state at 3, so we must continue traversing.

3-4 Constant Pressure Heating

We know

$P_3 = P_4 = 3572$ kPa

This now fixes state 3 and allows us to calculate the temperature from

$$T_3 = \frac{P_3 V_3}{mR} = \frac{(3572)(1.23 \text{ x } 10^{-4})}{(1.53 \text{ x } 10^{-3})(0.287)} = 1001 \text{ K}$$

We can now go to the air tables and find

$u_3 = 759.42$ kJ/kg and $\phi_3 = 2.9683$ kJ/(kg·K)

4-5 Isentropic expansion

Again we use the v_r from the air tables for these process, then

$$v_{r5} = v_{r4}\left(\frac{V_5}{V_4}\right) = (4.761)\left(\frac{9.23 \times 10^{-4}}{2.09 \times 10^{-4}}\right) = 21.03$$

Interpolating from the air tables we get

$T_5 = 1062$ K and $u_5 = 812.32$ kJ/kg and $\phi_5 = 3.0366$ kJ/(kg·K)

and from the ideal gas equation

$$P_5 = \frac{mRT_5}{V_5} = \frac{(1.53 \times 10^{-3})(0.287)(1062)}{(9.23 \times 10^{-4})} = 505 \text{ kPa}$$

The engine thermal efficiency is given by

$$\eta_{th} = \frac{W_{net}}{Q_{in}}$$

where W_{net} and Q_{in} can be either on a per cylinder basis or an engine basis, as long as they are both on the same basis. Using a per cylinder basis we have

$$W_{net} = W_{1\text{-}2} + W_{3\text{-}4} + W_{4\text{-}5}$$

where

$$W_{1\text{-}2} = m(u_1 - u_2) = (1.53 \times 10^{-3})(224.85 - 505.57) = -0.4295 \text{ kJ}$$

$$W_{3\text{-}4} = P_3(V_4 - V_3) = (3572)(2.09 \times 10^{-4} - 1.23 \times 10^{-4})$$

$$= 0.30719 \text{ kJ}$$

$$W_{4\text{-}5} = m(u_4 - u_5) = (1.53 \times 10^{-3})(1392.7 - 812.32) = 0.88798 \text{ kJ}$$

and

$$W_{net} = (-0.4295) + (0.30719) + (0.88798) = 0.76567 \text{ kJ}$$

Then

$$Q_{in} = Q_{2\text{-}3} + Q_{3\text{-}4}$$

where

$$Q_{2\text{-}3} = m(u_3 - u_2) = (1.53 \times 10^{-3})(759.42 - 502.57)$$

$$= 0.39298 \text{ kJ}$$

$$Q_{3\text{-}4} = m(u_4 - u_3) + W_{3\text{-}4} = (1.53 \times 10^{-3})(1392.7 - 759.42) + 0.30719$$

$$= 1.2761 \text{ kJ}$$

and

$$Q_{in} = 0.39298 + 1.2761 = 1.6691 \text{ kJ}$$

Then

$$\eta_{th} = \frac{0.76567}{1.6691} = 0.459$$

The engine power output is given by

$$\dot{W}_{net} = N_{cyl}\left(W_{net}\right)_{cyl}(rps)(1/2)$$

$$= (2)(0.76567)(1300/60)(0.5)$$

$$= 16.6\ kW = 22.2\ hp$$

The engine MEP is given by

$$MEP = \frac{\left(W_{net}\right)_{cyl}}{\left(V_{disp}\right)_{cyl}} = \frac{0.76567}{(8\ x\ 10^{-4})} = 957\ kPa$$

To calculate the turbine power we need to first determine the turbine work, given by

$$w_t = h_5 - h_{out}$$

To get h_5 we simply go to the air tables and T_5 and find

$h_5 = 1117.2$ kJ/kg

To determine h_{out}, we must first know T_{out}, but we do know that the turbine exhausts to 100 kPa and it is isentropic, then

$$\phi_{out} = \phi_5 + R \cdot \ln\left(\frac{P_{out}}{P_5}\right) = 3.0366 + (0.287)\ln\left(\frac{100}{505}\right)$$

$$= 2.5718$$

which gives from the air tables

$h_{out} = 712.62$ kJ/kg

Then

$$w_t = 1117.2 - 712.62 = 404.58\ kJ/kg$$

To convert this into a power we need to calculate the mass flow rate through the turbine. Assuming all four cylinders exhaust into the same cylinder we can write

$$\dot{m}_t = N_{cyl}(0.85)m_{cyl}(rps)(1/2)$$

$$= (2)(0.85)(1.53\ x\ 10^{-3})(1300/60)(0.5)$$

$$= 0.0282\ kg/s$$

and our turbine power is then

$$\dot{W}_t = \dot{m}_t w_t = (0.0282)(404.58) = 11.4\ kW$$