ME 201
Thermodynamics

Homework #1: Conservation of Mass
Solution

1. Describe mass conservation for a real world system such as the human body or a jet aircraft engine. (5 pts)

Solution: Various answers possible

2. During an attack, the asthma sufferer actually accumulates air in his/her lungs causing great difficulty in respiration. A critical point comes when the accumulated air fills 75% of the lung capacity at which time medication becomes essential. The volume of lungs may be taken to be 0.006 m^3 and the air intake is 1.26 x 10^{-6} kg/s with only 85% of the intake exhausted during an attack. Determine the time it takes from the onset of an attack to the critical point when medication becomes essential. You may take the density of air to be 1.3 kg/m^3. (10 pts)

Solution: Let's begin by writing our complete conservation of mass equation,

$$\frac{dm_{sys}}{dt} = \sum_{inflows} \dot{m}_{in} - \sum_{outflows} \dot{m}_{out}$$

We recognize that we have only one inflow (air intake) and just one outflow (exhaust). We can explode the derivative in to a difference and re-write the equation as

$$\frac{m_{sys}(\Delta t) - m_{sys}(0)}{\Delta t} = \dot{m}_{in} - \dot{m}_{out}$$

Solving for Δt

$$\Delta t = \frac{m_{sys}(\Delta t) - m_{sys}(0)}{\dot{m}_{in} - \dot{m}_{out}}$$

Now evaluating our masses and mass flows

$$\dot{m}_{in} = 1.26 \text{ x } 10^{-6} \text{ kg / s}$$

$$\dot{m}_{out} = (0.85)\dot{m}_{in} = (0.85)\left(1.26 \text{ x } 10^{-6}\right) = 1.07 \text{ x } 10^{-6} \text{ kg / s}$$

$\dot{m}_{sys}(0) = 0$ kg (assume no air initially stored in the lungs)

$\dot{m}_{sys}(\Delta t) = 75\%$ lung capacity

$= 75\% \times$ density of air $\times$ volume capacity of lungs

$= (0.75)(1.3)(0.006) = 5.85 \text{ x } 10^{-3}$ kg

Now substituting we find

$$\Delta t = \frac{5.85 \text{ x } 10^{-3} - 0}{1.26 \text{ x } 10^{-6} - 1.07 \text{ x } 10^{-6}} = 30{,}952 \text{ s} = 8.6 \text{ hr}$$