ME 201
Thermodynamics

Homework 18
Solutions

1. Determine the work per mass output of an adiabatic turbine with isentropic efficiency 0.83 that has a steam input of 15 MPa and 650°C and an outlet pressure of 50 kPa.

Solution:

The actual work will be calculated from the isentropic efficiency by

$$w_{act} = \eta_s w_{ideal}$$

Hence we need to calculate the ideal work. We begin by setting up our table and working the 1st law problem

System Type: Control Volume (Turbine)
Working Fluid: Steam (compressible Substance)
Process: Isentropic (ideal) Adiabatic (actual) with $\eta_s = 0.83$

State 1	State 2s	State 2a
$T_1 = 650°C$	$T_{2s} = \mathit{81.32\,°C}$	$T_{2a} = \mathit{81.32\,°C}$
$P_1 = 15$ MPa	$P_{2s} = 50$ kPa	$P_{2a} = 50$ kPa
$h_1 =$ *3712.1 kJ/kg*	$h_{2s} =$ ***2372.4 kJ/kg***	$h_{2a} =$ **2600 kJ/kg**
$s_1 =$ *6.8233 kJ/(kg·K)*	$s_{2s} =$ **6.8233 kJ/(kg·K)**	$s_{2a} =$ ***7.4649 kJ/(kg·K)***
phase: sup.vap.	phase: *2 phase with* $\boldsymbol{x_{2s} = 0.882}$	phase: *2 phase with* $\boldsymbol{x_{2a} = 0.98}$

Italicized values are from steam tables. **Bold** values are calculated.

State1: Fixed
State2s: UNKNOWN
State2a: UNKNOWN
$\dot{Q} = 0$
$\dot{W}_{sh} =$ UNKNOWN
1st Law: $\dot{m}[h_1 - h_2] = \dot{W}_{sh}$

Approach: To fix the state2s we will use our process description which will gives us our entropy at state 2s. The remaining properties can then be determined from the steam tables. The ideal power is then calculated from the 1st law. The definition of isentropic efficiency will be used to calculate the actual power. The first law can then be used to calculate the actual exit enthalpy, which will fix state 2a and the properties can then be determined from the steam tables.

We start by completing our information for state 1. Going to the steam tables we find that we have superheated vapor with

$h_1 = 3712.1$ kJ/kg and $s_1 = 6.8233$ kJ/(kg·K)

Since we have an isentropic process the s at state 2s is given by

$s_{2s} = s_1 = 6.8222$ kJ/(kg·K)

Then from the steam tables at 50 kPa we find that we have a two phase mixture with quality

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{6.8224 - 1.0912}{7.5931 - 1.0912} = 0.882$$

The enthalpy is then

$$h_{2s} = h_f + x_{2s} h_{fg} = 340.54 + (0.88)(2304.7) = 2372.4 \text{ kJ/kg}$$

and the temperature is

$$T_{2s} = 81.32°\text{C}$$

Next we will calculate our ideal specific work from the 1st law

$$w_{ideal} = h_1 - h_{2s} = 3712.1 - 2372.4 = 1339.7 \text{ kJ/kg}$$

Then our actual specific work is

$$w_{act} = \eta_s w_{ideal} = (0.83)(1339.7) = 1112 \text{ kJ/kg}$$

Our actual exit enthalpy is

$$h_{2a} = h_1 - w_{act} = 3712.3 - 1112 = 2600 \text{ kJ/kg}$$

We now go to the steam tables and find that at state 2a we have two phase mixture with quality

$$x_{2a} = \frac{h_{2a} - h_f}{h_g - h_f} = \frac{2600.2 - 340.54}{2304.7} = 0.98$$

The entropy is then

$$s_{2a} = s_f + x_{2a} s_{fg} = 1.0912 + (0.98)(6.5019) = 7.4658 \text{ kJ/(kg} \cdot \text{K)}$$

and the temperature is

$$T_{2s} = 81.32°\text{C}$$

2. Refrigerant-134a enters an adiabatic pump at 90 psia and 60°F and exits at 120 psia and 62°F. For a flow rate of 12 kg/s determine

   (a) the actual power required
   (b) the isentropic efficiency

<u>Solution:</u>
We begin by setting up our table and working the 1st law problem. We convert our temperatures and pressures into SI units.

System Type: Control Volume (Pump)
Working Fluid: R-134a (incompressible Substance)
Process: Isentropic (ideal) Adiabatic (actual) with η_s = ??

State 1	State 2s	State 2a
T_1 = 15.9°C	T_{2s} = **15.9°C**	T_{2a} = 17°C
P_1 = 619.2 kPa	P_{2s} = 825.6 kPa	P_{2a} = 825.6 kPa
phase: *sub.liq.* (incomp. sub.)	phase: *sup.vap.*	phase: *sub. liq.*

Bold values are calculated.

State1: Fixed
State2s: UNKNOWN
State2a: Fixed
$\dot{Q} = 0$
$\dot{W}_{sh}$ = UNKNOWN
1st Law: $\dot{m}\left[h_1 - h_2\right] = \dot{W}_{sh}$

Approach: Since state 2a is fixed we can apply the first law we can obtain the actual power. To fix the state2s we will use our process description which will gives us our other property at state 2s. The ideal work is then calculated from the 1st law. Then the isentropic efficiency will be calculated from its definition.

We start by recognizing that we have subcooled liquids, but no compressed liquid tables for R-134a. Hence we will have to treat these liquids as incompressible substances. Then

$$h_1 - h_{2a} = c_{p,avg}\left(T_1 - T_{2a}\right) - v_{avg}\left(P_1 - P_{2a}\right)$$

We evaluate our properties at 16.4°C. Then

$\rho_{avg} = \rho_f(@16.4°C) = 1264.9\ kg/m^3$
$v_{avg} = 1/\rho_{avg} = 1/1264.9 = 0.000791\ m^3/kg$
$c_{P,avg} = c_P(@16.4°C) = 1.371\ kJ/(kg{\cdot}K)$

So that

$$h_1 - h_{2a} = (1.371)(15.9 - 17) - (0.000791)(619.2 - 825.6) = -1.686 \text{ kJ/kg}$$

and

$$\dot{W}_{act} = \dot{m}(h_1 - h_{2a}) = (12)(-1.686) = 20.24 \text{ kW required}$$

For the ideal power we have

$$\Delta s = 0$$

which becomes

$$\Delta s = c_{p,avg} \ln(T_{2s}/T_1) = 0$$

and gives

$$T_{2s} = T_1 = 15.9°C$$

Then

$$h_1 - h_{2s} = v_{avg}(P_1 - P_{2a}) = (0.000791)(619.2 - 825.6) = -0.1632$$

so that

$$\dot{W}_{ideal} = \dot{m}(h_1 - h_{2a}) = (12)(-0.1632) = 1.96 \text{ kW required}$$

Finally

$$\eta_s = \frac{1.96}{20.24} = 0.097 \text{ or } 9.7\%$$