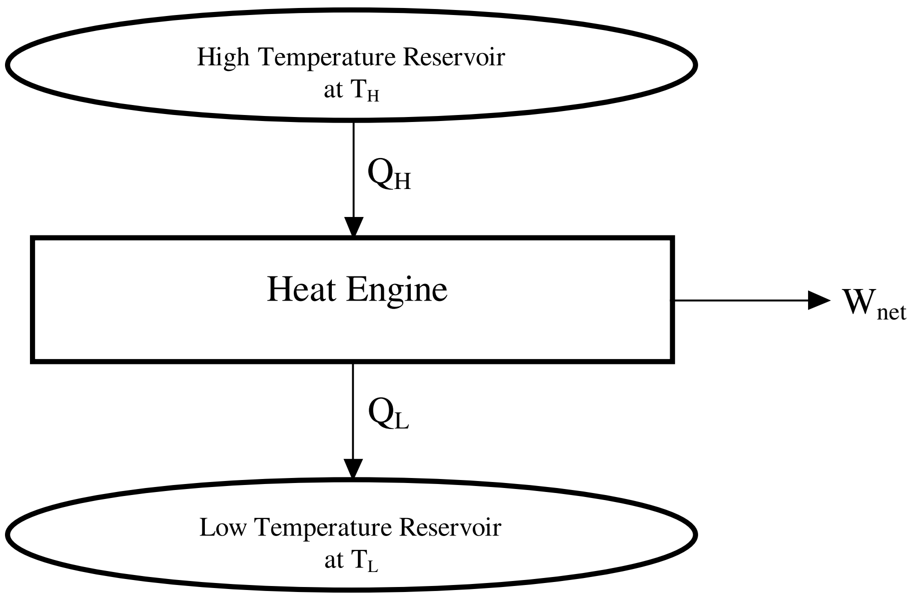
ME 201
Thermodynamics

Homework 16 Solution

1. Consider a power plant that is producing 1 MW of electric power as it operates with a high temperature of 1800 K and a low temperature of 290 K. If we can sell the electric power for $0.04 per kW·hr, and the heat transfer from the high temperature reservoir costs $0.0075 per kW·hr, what is the maximum income per year the plant can generate?

Solution:

We begin with our interaction diagram for a heat engine.



Our appropriate equations are:

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_H}$$

$$\dot{Q}_H = \dot{W}_{net} + \dot{Q}_L$$

$$\eta_{Carnot} = 1 - \frac{T_L}{T_H}$$

Our income is given by

Income = (Electric Energy Generated)(0.04) - (Heat Energy Consumed)(0.0075) where

Electric Energy Generated = $\dot{W}_{net}$ x # of hours in a year

= (1000 kW)(24 x 365) = 8,760,000 kW·hr

Heat Energy Consumed = $\dot{Q}_H \times$ # of hours in a year

We can determine $\dot{Q}_H$ from our thermal efficiency. The thermal efficiency is calculated from our Carnot equation or

$$\eta_{Carnot} = 1 - \frac{290}{1800} = 0.839$$

Our heat transfer rate is then

$$\dot{Q}_H = \frac{\dot{W}_{net}}{\eta_{th}} = \frac{1000}{0.839} = 1192 \text{ kW}$$

and our annual income is

Income = (8,760,000)(0.04) - (1192)(24)(365)(0.0075) = $272,100

2. It is clear that we can improve the efficiency of a heat engine by lowering the temperature to which the heat rejection occurs. One way to achieve this is to use a refrigerator to produce a low temperature heat reservoir. It is proposed to do this to the power plant of problem #1, lowering the low temperature reservoir to 240 K. If the power plant is still to produce 1 MW of electric power, what is the maximum income per year the plant can generate? Remember to include the power requirements of the refrigerator in this calculation. The high temperature heat reservoir for the refrigerator is at 290 K.

Solution:

We note that our income is still given by

Income = (Electric Energy Generated)(0.04) - (Heat Energy Consumed)(0.0075)

but now

Electric Energy Generated = $\left(\dot{W}_{HE} - \dot{W}_{refrig}\right)$ x # of hours in a year

Also our Heat Energy Consumed has changed. Beginning with this we now find

$$\eta_{Carnot} = 1 - \frac{240}{1800} = 0.867$$

Our heat transfer rate is then

$$\dot{Q}_H = \frac{\dot{W}_{net}}{\eta_{th}} = \frac{1000}{0.867} = 1154 \text{ kW}$$

The power requirement of our refrigerator is given by

$$\dot{W}_{net} = \frac{\dot{Q}_L}{COP}$$

where

$$COP = \frac{1}{\frac{T_H}{T_L} - 1} = \frac{1}{\frac{290}{240} - 1} = 4.8$$

and the heat transfer rate is given by

$$\dot{Q}_L = \left(\dot{Q}_H - \dot{W}_{net}\right)_{HE} = (1154 - 1000) = 154 \text{ kW}$$

Then

$$\dot{W}_{net} = \frac{154}{4.8} = 32.1 \text{ kW}$$

Our annual income is then

Income = (1000-32.1)(24)(365)(0.04) - (1154)(24)(365)(0.0075) = $263,400

so it is not worth using the refrigerator.