# ME 201
# Thermodynamics

## Homework 14 Solution

1. Consider the Carnot cycle occurring in a piston-cylinder device containing refrigerant-134a with operating conditions given below:

**Process A:** Isothermal heat addition at $T_H = 30°C$ to convert saturated liquid to saturated vapor
**Process B:** Isentropic and adiabatic expansion to $T_L = -20°C$
**Process C:** Isothermal heat removal at -20°C
**Process D:** Isentropic and adiabatic compression back to the initial state

With R-134a as the working fluid for this cycle calculate the thermal efficiency using

$$\eta_{th} = \frac{W_{net}}{Q_{in}}$$

Compare this to the ideal Carnot cycle efficiency given by

$$\eta_{Carnot} = 1 - \frac{T_L}{T_H}$$

**Solution:**
We shall make our calculations process by process

Process A
Working Fluid: R-134a(compressible)
System: Closed System
Process: Isothermal

| State 1 | State 2 |
| --- | --- |
| $T_1 = 30°C$ | $T_2 = 30°C$ |
| $P_1 = \mathit{770.64\ kPa}$ | $P_2 = \mathit{770.64\ kPa}$ |
| $v_1 = \mathit{0.0008421\ m^3/kg}$ | $v_2 = \mathit{0.026622\ m^3/kg}$ |
| $u_1 = \mathit{92.93\ kJ/kg}$ | $u_2 = \mathit{246.14\ kJ/kg}$ |
| $s_1 = \mathit{0.34789\ kJ/(kg\ K)}$ | $s_2 = \mathit{0.91879\ kJ/(kg\ K)}$ |
| phase: sat.liq. | phase: sat.vap. |

*italicized* values from tables
Initial State: Fixed

Final State: Fixed

$W_{sh} = 0$

Q = ????

$W_{bnd}$ = ????

1st Law: $u_2 - u_1 = q - w_{bnd}$

Pdv: $w_{bnd} = P(v_2 - v_1)$

Both states are fixed, so we can go to the R-134a tables and look up the remaining properties. Since our process is both isothermal and isobaric due to the phase change, the boundary work is given by

$$w_A = P_1(v_2 - v_1) = (770.64)(0.026622 - 0.0008421)$$
$$= 19.867 \text{ kJ/kg}$$

The heat transfer will be given by the 1st law, or

$$q_A = w_A + u_2 - u_1 = 19.867 + 246.14 - 92.93$$
$$= 173.077 \text{ kJ/kg}$$

Process B

Working Fluid: R-134a(compressible)

System: Closed System

Process: Isentropic and Adiabatic

| State 2 | State 3 |
|---|---|
| $T_2 = 30°C$ | $T_3 = -20°C$ |
| $P_2$ = *770.64 kPa* | $P_3$ = *132.82 kPa* |
| $v_2$ = *0.026622 m³/kg* | $v_3$ = *0.142611 m³/kg* |
| $u_2$ = *246.14 kJ/kg* | $u_3$ = *212.6639 kJ/kg* |
| $s_2$ = *0.91879 kJ/(kg K)* | $s_3$ = **0.91879 kJ/(kg K)** |
| phase: sat.vap. | phase: *2 phase, x = 0.968* |

*italicized* values from tables

Initial State: Fixed

Final State: UNKNOWN

$W_{sh} = 0$

$Q_B = 0$

$W_{bnd}$ = ????

1st Law: $u_3 - u_2 = -w_{bnd}$

Since our process is isentropic we have

$s_3 = s_2 = 0.91879$ kJ/(kg·K)

Going to the tables at -20°C, we find

$s_f = 0.10463$ kJ/(kg·K) and $s_g = 0.94564$ kJ/(kg·K)

so that we have a two phase mixture with

$$x_3 = \frac{0.91879 - 0.10463}{0.94564 - 0.10463} = 0.968$$

The remaining properties can then be calculated. The boundary work is given by the first law

$$w_B = u_2 - u_3 = 246.14 - 212.6639$$
$$= 33.47 \text{ kJ/kg}$$

Process C
Working Fluid: R-134a(compressible)
System: Closed System
Process: Isothermal

| State 3 | State 4 |
|---|---|
| $T_3$ = -20°C | $T_4$ = -20°C |
| $P_3$ = 132.82 kPa | $P_4$ = |
| $v_3$ = 0.142611 $m^3$/kg | $v_4$ = |
| $u_3$ = 212.6639 kJ/kg | $u_4$ = |
| $s_3$ = 0.91879 kJ/(kg K) | $s_4$ |
| phase: 2 phase, x = 0.968 | phase: |

*italicized* values from tables
Initial State: Fixed
Final State: Fixed
$W_{sh} = 0$
$q_C$ = ?????
$W_{bnd}$ = ????
1st Law: $u_4 - u_3 = q - w_{bnd}$
Pdv: $w_{bnd} = P(v_4 - v_3)$

Immediately, we see that we do not have enough information to fix State 4. We will continue on to Process D and return later to this process.

Process D
Working Fluid: R-134a(compressible)
System: Closed System
Process: Isentropic and Adiabatic

| State 4 | State 1 |
|---|---|
| $T_4$ = -20°C | $T_1$ = 30°C |
| $P_4$ = *132.82 kPa* | $P_1$ = 770.64 kPa |
| $v_4$ = *0.043127 $m^3$/kg* | $v_1$ = 0.0008421 $m^3$/kg |
| $u_4$ = *81.34492 kJ/kg* | $u_1$ = 92.93 kJ/kg |
| $s_4$ = **0.34789 kJ/(kg K)** | $s_1$ = 0.34789 kJ/(kg K) |
| phase: *2 phase, x = 0.289* | phase: sat.liq. |

*italicized* values from tables
Initial State: UNKNOWN
Final State: Fixed

$W_{sh} = 0$
$q_D = 0$
$W_{bnd} = ????$
$1^{st}$ Law: $u_1 - u_4 = -w_{bnd}$

Since our process is isentropic we recognize that

$s_4 = s_1 = 0.34789$ kJ/(kg K)

Going to the tables at -20°C, we find

$s_f = 0.10463$ kJ/(kg·K) and $s_g = 0.94564$ kJ/(kg·K)

so that we have a two phase mixture with

$$x_4 = \frac{0.34789 - 0.10463}{0.94564 - 0.10463} = 0.289$$

The remaining properties can then be calculated. We have a compressible substance the boundary work is given by the first law

$$w_D = u_4 - u_1 = 81.34492 - 92.93$$
$$= -11.585 \text{ kJ/kg}$$

Now returning to process C since our state 4 is now fixed.

Process C
Working Fluid: R-134a(compressible)
System: Closed System
Process: Isothermal

| State 3 | State 4 |
|---|---|
| $T_3 = -20°C$ | $T_4 = -20°C$ |
| $P_3 = 132.82$ kPa | $P_4 = 132.82$ kPa |
| $v_3 = 0.142611\ m^3/kg$ | $v_4 = 0.043127\ m^3/kg$ |
| $u_3 = 212.6639$ kJ/kg | $u_4 = 81.34492$ kJ/kg |
| $s_3 = 0.91879$ kJ/(kg K) | $s_4 = 0.34789$ kJ/(kg K) |
| phase: 2 phase, x = 0.968 | phase: 2 phase, x = 0.289 |

*italicized* values from tables
Initial State: Fixed
Final State: Fixed
$W_{sh} = 0$
$q_C = ?????$
$W_{bnd} = ????$
$1^{st}$ Law: $u_4 - u_3 = q - w_{bnd}$
Pdv: $w_{bnd} = P(v_4 - v_3)$

Since our process is both isothermal and isobaric due to the phase change, the boundary work is given by

$$w_C = P_3(v_4 - v_3) = (132.82)(0.043127 - 0.142611)$$
$$= -13.214 \text{ kJ/kg}$$

The heat transfer will be given by the 1st law, or

$$q_C = w_C + u_4 - u_3 = -13.214 + 81.34 - 212.66$$
$$= -144.53 \text{ kJ/kg}$$

Our thermal efficiency is given by

$$\eta_{th} = \frac{w_{net}}{q_{in}}$$

The net work is given by

$$w_{net} = w_A + w_B + w_C + w_D$$
$$= 19.867 + 33.47 + (-13.214) + (-11.585)$$
$$= 28.545 \text{ kJ/kg}$$

Since heat is only added during process A

$$q_{in} = q_A$$
$$= 173.077 \text{ kJ/kg}$$

So that

$$\eta_{th} = \frac{28.545}{173.077} = 0.1649$$

The Carnot cycle efficiency is

$$\eta_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{(-20 + 273)}{(30 + 273)}$$
$$= 0.165$$