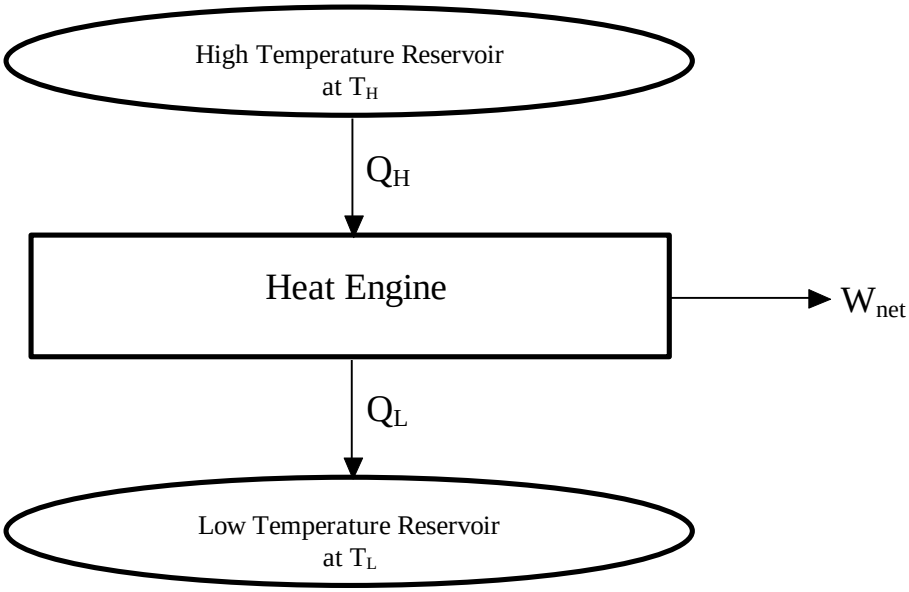
ME 201
Thermodynamics

Homework 11 Solutions

1. (10 pts) A Carnot heat engine produces power of 2.5 kW. It rejects heat to a river that is flowing at 2 kg/s, resulting in a temperature increase of 2°C. The average temperature of the river is 20°C. Determine

   (a) the heat transfer input required for the heat engine
   (b) the efficiency of the heat engine
   (c) the temperature at which heat transfer occurs to the engine

Solution:
We begin with our interaction diagram for a heat engine.



Our appropriate equations are:

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_H}$$

$$\dot{Q}_H = \dot{W}_{net} + \dot{Q}_L$$

$$\eta_{Carnot} = 1 - \frac{T_L}{T_H}$$

We note that we have 2 parameters given, $\dot{W}_{net} = 2.5\,\text{kW}$ and $T_L = 293\,\text{K}$, and four unknowns, $\dot{Q}_H, \dot{Q}_L, T_H,$ and η_{th} . Clearly with just three equations we have a poorly posed problem. In these cases we need to see if we can determine a heat transfer rate from a 1st law analysis on one of the heat reservoirs. For this problem the most likely candidate is the low temperature heat reservoir. Then setting up our 1st law problem structure

System: Control Volume
Working Fluid: Water (compressible substance)
Process: River (heat device)

State 1	State 2
T_1 = 19°C	T_2 = 21°C
P_1 = 100 kPa	P_2 = **100 kPa**
h_1 =	h_2 =
phase: *sub.liq.*	phase: *sub.liq.*

Bold values are calculated, *italicized* values from steam table.

State 1: fixed
State 2: UNKNOWN
Wsh = 0
$Q = Q_L$
1st law: $0 = \dot{m}_{in}h_{in} - \dot{m}_{out}h_{out} + \dot{Q}$

We note that if our river is a heat device then

$P_2 = P_1 = 100$ kPa

To determine the phase we go to the steam tables at 100 kPa and find a saturation temperature of 99.63°C, so that we have subcooled liquid. If we go to the compressed liquid table we find that our pressure is too low to be found on the table and so we must treat the water as an incompressible substance. Then our enthalpy change is given by

$h_{out} - h_{in} = c_{p,avg}(T_{out}-T_{in}) + v_{avg}(P_{out}-P_{in})$

But since our pressure change is zero

$h_{out} - h_{in} = c_{p,avg}(T_{out}-T_{in})$

Evaluating $c_{p,avg}$ at 20°C (293 K), we have

$h_{out} - h_{in} = (4.17)(2) = 8.34$ kJ/kg

and the heat transfer becomes

$$\dot{Q} = \dot{m}_{in}\left(h_{out} - h_{in}\right) = (2)(8.34) = 16.68\,\text{kW} = \dot{Q}_L$$

Now we can solve for $\dot{Q}_H$ from our energy balance equation or

$$\dot{Q}_H = 2.5 + 16.68 = 19.18\,\text{kW}$$

The thermal efficiency is then

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{2.5}{19.18} = 0.13$$

Solving for the high temperature heat reservoir temperature we have

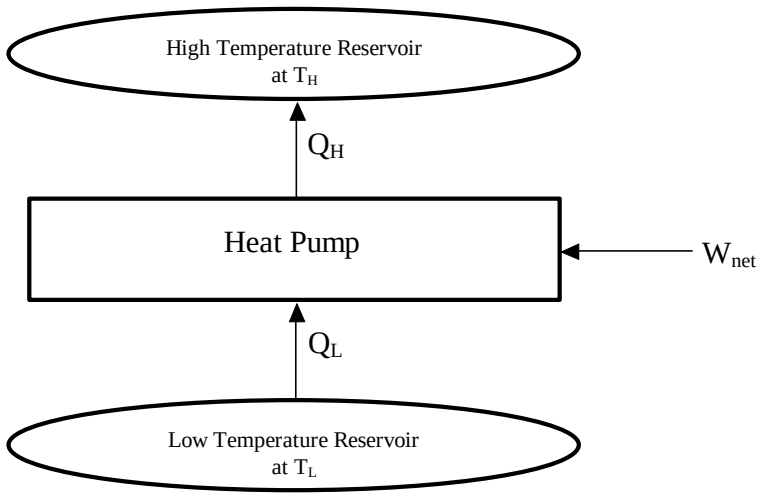
$$T_H = \frac{T_L}{1-\eta_{Carnot}} = \frac{293}{1-0.13} = 337\,\text{K}$$

2. (10 pts) A retired engineer living in Arizona is attempting to heat her home by using a heat pump. She has determined the minimum heat transfer rate needed to heat her home to be 50 kJ/h. She has the opportunity to purchase a heat pump that has a COP of 5 and requires 2.75 W of electric power to run. Should she buy this heat pump? If the inside of the house is to be

maintained at 70°F and the average low outside temperature is 40°F, how does the performance of the actual heat pump compare with the maximum possible performance?

Solution:

We begin with our interaction diagram for a heat pump.



Our appropriate equations are:

$$COP = \frac{\dot{Q}_H}{\dot{W}_{net}}$$

$$\dot{Q}_H = \dot{W}_{net} + \dot{Q}_L$$

$$COP_{Carnot} = \frac{1}{1 - \frac{T_L}{T_H}}$$

The first part of the problem simply requires us to evaluate whether the heat pump being considered actually can satisfy the engineer's need. To do this we will calculate the heat pumps heat transfer rate from

$$\dot{Q}_H = COP \cdot \dot{W}_{net} = (5)(2.75) = 13.75\ W$$

The required heat transfer rate is 50 kJ/h or converting to W

$$\dot{Q}_{H,req} = (50)(1000\ J/kJ)/(3600\ s/hr) = 13.89\ W$$

So that the proposed heat pump will NOT be adequate. The best performing heat pump would have the COP of a Carnot heat pump, so that

$$COP_{Carnot} = \frac{1}{1 - \frac{40 + 460}{70 + 460}} = 17.7$$

so there is much room for improvement in the proposed heat pump that has a COP of 5.