First Law Practice Problems Set #2

1. A student living in a 4m x 6m x 6m dormitory room turns on her 150 W fan before leaving the room on a summer day. The room is initially at 100 kPa and 15°C. Assuming that the room is adiabatic, determine the room temperature 10 hours later.

Solution:

System Type: Closed System
Working Fluid: Air (ideal gas)
Process: Constant Volume
Initial State: Fixed
Final State: UNKNOWN

\[ Q = 0 \]
\[ W_{\text{sh}} = 150 \text{ W} \]
\[ W_{\text{bnd}} = 0 \]
Conservation of Mass: \( m_1 = m_2 \)
1st Law: \( m(u_2-u_1) = W_{\text{sh}} \)

<table>
<thead>
<tr>
<th>State 1</th>
<th>State 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{1,1} = 15^\circ\text{C} = 288 \text{ K} )</td>
<td>( T_2 = 330.9 \text{ K} )</td>
</tr>
<tr>
<td>( P_1 = 100 \text{ kPa} )</td>
<td>( P_2 = 114.0 \text{ kPa} )</td>
</tr>
<tr>
<td>( u_1 = -7.22 \text{ kJ/kg} )</td>
<td>( u_1 = 23.81 \text{ kJ/kg} )</td>
</tr>
<tr>
<td>( V_1 = 4\text{x}6\text{x}6 = 144 \text{ m}^3 )</td>
<td>( V_1 = 144 \text{ m}^3 )</td>
</tr>
<tr>
<td>( v_1 = 0.8266 \text{ m}^3/\text{kg} )</td>
<td>( v_2 = 0.8266 \text{ m}^3/\text{kg} )</td>
</tr>
<tr>
<td>( m_1 = 174 \text{ kg} )</td>
<td>( m_2 = 174 \text{ lbm} )</td>
</tr>
</tbody>
</table>

*Italicized* values are from ideal gas relations or tables. **Bold** values are calculated.

Approach: To fix the final state we will use our process description along with the conservation of mass to determine the final specific volume. Then using the conservation of energy, we can determine the final specific internal energy. This will then fix the final state.

We start by determining our mass. Using the ideal gas law for the initial state we have

\[ v_1 = \frac{RT_1}{P_1} = \frac{(0.287)(288)}{(100)} = 0.8266 \text{ m}^3/\text{kg} \]
\[ m_1 = \frac{V_1}{v_1} = \frac{(144)}{(0.8266)} = 174 \text{ kg} \]

Then using conservation of mass
\[ m_2 = m_1 = 174 \text{ kg} \]

Since the volume and mass are both constant, the specific volume must also be constant or
\[ v_2 = 0.8266 \text{ m}^3/\text{kg} \]
Applying our conservation of energy we can write
\[ u_2 = u_1 + \frac{\dot{W}_{sh} \times \tau}{m} \]
where \( \tau \) is the time. Going to the ideal gas tables for air we find
\[ u_1 = -7.22 \text{ kJ/kg} \]
Then
\[ u_2 = -7.22 + \frac{(150 \times 10^{-3})(10 \times 3600)}{174} = 23.81 \text{ kJ/kg} \]
We can go to the ideal gas table and find a temperature of
\[ T_2 = 330.9 \text{ K} \]
This now allows use to use the ideal gas law to determine the pressure at state 2 or
\[ P_2 = \frac{RT_2}{\nu_2} = \frac{(0.287)(330.9)}{(0.8266)} = 114.9 \text{ kPa} \]

2. Ethylene Glycol at 90 kPa and 10°C enters a pump with an inlet area of 10 cm\(^2\) and exits at 140 kPa though an outlet area of 5 cm\(^2\). For a mass flow rate of 0.2 kg/s determine

   a. the exit temperature
   b. the required power

**Solution:**

**System Type:** Control Volume (Pump)
**Substance Type:** Incompressible
**Process:** Isentropic, PE are negligible. Include KE
**Inlet State:** Fixed
**Exit State:** UNKNOWN
\[ Q = 0 \]
\[ W_{sh} = \text{UNKNOWN} \]

1st Law: \( m \left( h_2 + \frac{v_2^2}{2} - h_1 - \frac{v_1^2}{2} \right) = \dot{W}_{sh} \)

<table>
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<tr>
<th>State 1</th>
<th>State 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_1 = 10^\circ \text{C} = 283 \text{ K} )</td>
<td>( T_2 = 283 \text{ K} )</td>
</tr>
<tr>
<td>( P_1 = 90 \text{ kPa} )</td>
<td>( P_2 = 140 \text{ kPa} )</td>
</tr>
<tr>
<td>( A_1 = 10 \text{ cm}^2 = 10^{-4} \text{ m}^2 )</td>
<td>( A_1 = 5 \text{ cm}^2 = 5 \times 10^{-4} \text{ m}^2 )</td>
</tr>
</tbody>
</table>

**Bold** values are calculated.

Approach: To fix the final state we will use our process description. The velocities can be calculated from continuity. The shaft work per mass can then be determined from the conservation of energy equation.

For an incompressible substance the entropy change is given by
\[ \Delta s = c_{p,avg} \ln \left( \frac{T_2}{T_1} \right) \]
So for an isentropic process, we must have
\[ T_2 = T_1 = 283 \text{ K} \]
We will calculate our velocities from continuity
\[ \dot{m} = \rho A \ddot{v} \]
Solving gives
\[ \ddot{v}_1 = \frac{\dot{m}}{\rho A} = \frac{0.2}{(893.69)(10^{-3})} = 0.22 \text{ m/s} \]
\[ \ddot{v}_2 = \frac{\dot{m}}{\rho A_2} = \frac{0.2}{(893.69)(5\times10^{-4})} = 0.45 \text{ m/s} \]
where the density for Ethylene Glycol (893.69 kg/m\(^3\)) comes from Table B.3SI. The shaft is then calculated from the first law (using the incompressible substance model for the enthalpy change)
\[ W_{sh} = \dot{m} \left( h_2 + \frac{\ddot{v}_2^2}{2} - h_1 - \frac{\ddot{v}_1^2}{2} \right) = \dot{m} \left( c_p,avg \left( T_2 - T_1 \right) + n_{avg} \left( P_2 - P_1 \right) + \frac{\ddot{v}_2^2}{2} - \frac{\ddot{v}_1^2}{2} \right) \]
\[ = (0.2) \left( \frac{1}{893.69}(140 - 90) + \left[ \frac{(0.45)^2}{2} - \frac{(0.22)^2}{2} \right] 10^{-3} \right) \]
\[ = 0.0112 \text{ kW} = 11.2 \text{ W} \]

3. Steam enters one side of a heat exchanger at 10 kPa and a quality of 0.95 and exits at 45°C. Cooling water enters the other side of the heat exchanger at 100 kPa and 20°C and exits at 35°C. Determine the ratio of the mass flow rate of steam to cooling water.

**Solution:**

**System Type:** Control Volume System
**Substance Type:** Water (phase change substance)
**Process:** Isobaric
**Inlet State:** Fixed
**Exit State:** Fixed
\[ Q = 0 \]
\[ W_{sh} = 0 \]

**Conserv. of Mass:** \[ \dot{m}_{in-1} = \dot{m}_{out-1} \quad \dot{m}_{in-2} = \dot{m}_{out-2} \]

**1st Law:** \[ \dot{m}_{in-1} h_{in-1} + \dot{m}_{in-2} h_{in-2} - \dot{m}_{out-1} h_{out-1} - \dot{m}_{out-2} h_{out-2} = 0 \]

<table>
<thead>
<tr>
<th>State In-1</th>
<th>State In-2</th>
<th>State Out-1</th>
<th>State Out-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{in-1} = 200 \text{°C} )</td>
<td>( T_{in-2} = 20 \text{°C} )</td>
<td>( T_{out} = 45 \text{°C} )</td>
<td>( T_{out} = 35 \text{°C} )</td>
</tr>
<tr>
<td>( P_{in-1} = 10 \text{ kPa} )</td>
<td>( P_{in-2} = 100 \text{ kPa} )</td>
<td>( P_{out} = 10 \text{ kPa} )</td>
<td>( P_{out} = 100 \text{ kPa} )</td>
</tr>
<tr>
<td>( h_{in-1} = 2693.93 \text{ kJ/kg} )</td>
<td>( h_{in-2} = 84.30 \text{ kJ/kg} )</td>
<td>( h_{out} = 189.29 \text{ kJ/kg} )</td>
<td>( h_{out} = 146.71 \text{ kJ/kg} )</td>
</tr>
<tr>
<td>phase: ( x = 0.95 )</td>
<td>phase: sub.liq.</td>
<td>phase: sub.liq.</td>
<td>phase: sub.liq.</td>
</tr>
</tbody>
</table>

*Italicized* values are from steam tables. *Bold* values are calculated.

Approach: We begin by evaluating the properties at all states by using the steam tables. Substituting the enthalpies into the 1st law the mass flow rate ratio can be determined.
For the steam inlet we have
\[ h_{\text{in-1}} = h_f + x_{\text{in-1}}h_{fg} = 852.082 + (0.95)(1938.79) = 2693.93 \text{ kJ/kg} \]
For the steam outlet we have subcooled liquid, so we will use
\[ h_{\text{out-2}} = h_f + v_f[P-P_{\text{sat}}] = 188.29 + (0.001010)(10-9.5914) = 189.29 \text{ kJ/kg} \]
Similarly for the cooling water
\[ h_{\text{in-2}} = h_f + v_f[P-P_{\text{sat}}] = 84.2043 + (0.001000)(100-2.3376) = 84.30 \text{ kJ/kg} \]
\[ h_{\text{out-2}} = h_f + v_f[P-P_{\text{sat}}] = 146.63 + (0.001006)(100-5.629) = 146.71 \text{ kJ/kg} \]
Solving for the mass flow rate ratio from the first law
\[ \frac{\dot{m}_1}{\dot{m}_2} = \frac{h_{\text{in-2}} - h_{\text{out-2}}}{h_{\text{out-1}} - h_{\text{in-2}}} = \frac{84.30 - 146.71}{189.29 - 2693.93} = 0.0249 \]