

# ME 201

## Thermodynamics

### Solutions First Law Practice Problems

1. Consider a balloon that has been blown up inside a building and has been allowed to come to equilibrium with the inside temperature of 25°C and inside pressure of 100 kPa. The diameter of the balloon is measured and found to be 0.13 m. The balloon is then taken outside and allowed to come to equilibrium with the outside temperature of -5°C and outside pressure of 100 kPa. Determine the boundary work, heat transfer, and final balloon diameter.

**Solution:**

System Type: Closed System

Substance Type: Ideal gas

Process: Isobaric

Initial State: Fixed

Final State: Fixed

Q = UNKNOWN

$W_{sh} = 0$

$W_{bnd} = \text{UNKNOWN}$

Conservation of Mass:  $m_1 = m_2$

1st Law:  $m(u_2 - u_1) = Q - W_{bnd}$

Boundary Work:  $P(v_2 - v_1)$

<b>State 1</b>	<b>State 2</b>
$T_1 = 25^\circ\text{C} = 298 \text{ K}$	$T_2 = -5^\circ\text{C} = 268 \text{ K}$
$P_1 = 100 \text{ kPa}$	$P_2 = 100 \text{ kPa}$
$D_1 = 0.13 \text{ m}$	$D_2 = \mathbf{0.1255 \text{ m}}$
$V_1 = 1.150 \times 10^{-3} \text{ m}^3$	$V_2 = 1.034 \times 10^{-3} \text{ m}^3$
$m_1 = 1.345 \times 10^{-3} \text{ kg}$	$m_2 = 1.345 \times 10^{-3} \text{ kg}$
$u_1 = 209.06 \text{ kJ/kg}$	$u_2 = 191.17 \text{ kJ/kg}$
$\phi_1 = 1.6783 \text{ kJ/(kg K)}$	$\phi_2 = 1.588 \text{ kJ/(kg K)}$
$v_1 = 0.855 \text{ m}^3/\text{kg}$	$v_2 = 0.769 \text{ m}^3/\text{kg}$

*Italicized values are from ideal gas relations. **Bold** values are calculated.*

Approach: We begin by evaluating the properties at state 1 and state 2 by using the air tables and the ideal gas law. We can then use our boundary work equation to calculate the boundary work. Finally, we use the conservation of energy to determine the heat transfer.

We start by determining our total volume at state 1. The volume of a sphere is given by

$$V_1 = \frac{4}{3}\pi R^3 = 1.150 \times 10^{-3} \text{ m}^3$$

Next we determine our mass. Using the ideal gas law we have

$$m_1 = \frac{P_1 V_1}{RT_1} = \frac{(100)(1.150 \times 10^{-3})}{(0.287)(298)} = 1.345 \times 10^{-3} \text{ kg}$$

$$v_1 = \frac{V_1}{m_1} = \frac{(1.150 \times 10^{-3})}{(1.345 \times 10^{-3})} = 0.855 \text{ m}^3/\text{kg}$$

We can go to the air tables and find

$$u_1 = 209.06 \text{ kJ/kg} \quad \phi_1 = 1.6783 \text{ kJ}/(\text{kg}\cdot\text{K})$$

$$u_2 = 191.17 \text{ kJ/kg} \quad \phi_2 = 1.5888 \text{ kJ}/(\text{kg}\cdot\text{K})$$

The specific volume at state 2 must be given by the ideal gas law, so that

$$v_2 = \frac{RT_2}{P_2} = \frac{(0.287)(268)}{(100)} = 0.769 \text{ m}^3/\text{kg}$$

The total final volume is then

$$V_2 = m_2 v_2 = (1.345 \times 10^{-3})(0.769) = 1.034 \times 10^{-3} \text{ m}^3$$

The final diameter is

$$D_2 = 2 \left[ \frac{3V_2}{4\pi} \right]^{1/3} = 0.1255 \text{ m}$$

The boundary work is calculated

$$W_{\text{bnd}} = P(V_2 - V_1) = (100)(1.034 \times 10^{-3} - 1.150 \times 10^{-3}) = -0.0116 \text{ kJ}$$

and the heat transfer is

$$Q = m(u_2 - u_1) + W_{\text{bnd}} = (1.345 \times 10^{-3})(191.17 - 209.06) + (-0.0116) \\ = -0.0357 \text{ kJ}$$

2. Air at 1800 K and 800 kPa enters an ideal turbine at 2.3 kg/s. The power output required of this turbine is 700 kW. Determine the exhaust temperature and pressure.

**Solution:**

System Type: Control Volume System

Substance Type: Ideal gas

Process: Isentropic

Initial State: Fixed

Final State: Unknown

$$Q = 0$$

$$W_{sh} = 700 \text{ kW}$$

Conservation of Mass:  $\dot{m}_2 = \dot{m}_1 = 2.3 \text{ kg/s}$ 

$$1\text{st Law: } \dot{m}(h_2 - h_1) = -\dot{W}_{sh}$$

<b>State 1</b>	<b>State 2</b>
$T_1 = 1800 \text{ K}$	$T_2 = 1552 \text{ K}$
$P_1 = 800 \text{ kPa}$	$P_2 = \mathbf{424 \text{ kPa}}$
$h_1 = 2003.3 \text{ kJ/kg}$	$h_2 = \mathbf{1698.95 \text{ kJ/kg}}$
$\phi_1 = 3.6684 \text{ kJ/(kg K)}$	$\phi_2 = 3.4864 \text{ kJ/(kg K)}$

*Italicized* values are from ideal gas relations. **Bold** values are calculated.

Approach: We begin by evaluating the properties at state 1 by using the air tables and the ideal gas law. We can then use the conservation of energy to determine the enthalpy at state 2. The air tables will give  $T_2$ . Finally, the isentropic relation will give the exit pressure

At state 1 we can go to the air tables and find

$$h_1 = 2003.3 \text{ kJ/kg} \quad \phi_1 = 3.6684 \text{ kJ/(kg}\cdot\text{K)}$$

From the first law we solve for  $h_2$

$$h_2 = h_1 - \frac{\dot{W}_{sh}}{\dot{m}} = 2003.3 - \frac{700}{2.3} = 1698.95 \text{ kJ/kg}$$

The air tables can then give

$$T_2 = 1552 \text{ K} \quad \phi_2 = 3.4864 \text{ kJ/(kg}\cdot\text{K)}$$

The isentropic relation is

$$\Delta s = 0 = \phi_2 - \phi_1 - R \cdot \ln\left(\frac{P_2}{P_1}\right)$$

Solving for  $P_2$  gives

$$P_2 = P_1 \cdot \exp\left\{\frac{\phi_2 - \phi_1}{R}\right\} = (800) \exp\left\{\frac{3.4864 - 3.6684}{0.287}\right\} = 424 \text{ kPa}$$

3. Refrigerant-134a as saturated liquid at 32°C enters a valve and exits at 0.16 MPa.  
What is the fluid phase at the exit?

**Solution**

System Type: Control Volume System

Substance Type: Compressible

Process: Isenthalpic

Initial State: Fixed

Final State: Unknown

$$Q = 0$$

$$W_{sh} = 0$$

$$1st\ Law: h_2 - h_1 = 0$$

<b>State 1</b>	<b>State 2</b>
$T_1 = 32^\circ C$	$T_2 = -15.62^\circ C$
$P_1 = 0.81528\ MPa$	$P_2 = 0.16\ MPa$
$h_1 = 94.39\ kJ/kg$	$h_2 = \mathbf{94.39\ kJ/kg}$
Phase: sat.liq.	Phase: 2 phase, $x_2 = 0.31$

*Italicized* values are from R-134a tables. **Bold** values are calculated.

Approach: We begin by evaluating the properties at state 1 by using the refrigerant tables. We can then use the conservation of energy to determine the enthalpy at state 2. The refrigerant tables will give the fluid phase.

At state 1 we can go to the refrigerant tables and find

$$h_1 = 94.39\ kJ/kg \quad P_1 = 0.81528\ MPa$$

From the first law we solve for  $h_2$

$$h_2 = h_1 = 94.39\ kJ/kg$$

At 0.16 MPa, we find from the tables

$$h_f = 29.78\ kJ/kg \quad h_g = 237.97\ kJ/kg$$

Since  $h_2$  is between these two values we have a two phase mixture at  $-15.62^\circ C$  and with quality

$$x_2 = \frac{h_2 - h_f}{h_{fg}} = \frac{94.39 - 29.78}{208.18} = 0.31$$

4. A tank contains Refrigerant-134a as saturated vapor at 100 kPa. There is heat transfer to the tank of 66 kJ/kg. Determine the final temperature and pressure.

**Solution:**

System Type: Closed System  
 Substance Type: Compressible  
 Process: Isotropic  
 Initial State: Fixed  
 Final State: Unknown  
 $q = 66 \text{ kJ/kg}$   
 $W_{sh} = 0$   
 $W_{bnd} = 0$   
 Conservation of Mass:  $m_1 = m_2$   
 1st Law:  $u_2 - u_1 = q$

<b>State 1</b>	<b>State 2</b>
$T_1 = -26.43^\circ\text{C}$	$T_2 = 60^\circ\text{C}$
$P_1 = 100 \text{ kPa}$	$P_2 = 140 \text{ kPa}$
$u_1 = 212.18 \text{ kJ/kg}$	$u_2 = \mathbf{278.18 \text{ kJ/kg}}$
$v_1 = 0.1917 \text{ m}^3/\text{kg}$	$v_2 = \mathbf{0.1917 \text{ m}^3/\text{kg}}$
phase: sat.vap.	phase: <i>sup.vap.</i>

*Italicized values are from tables. Bold values are calculated.*

Approach: We begin by evaluating the properties at state 1 by using the refrigerant tables. We can then use isotropic process to determine the specific volume at state 2. Next, we use the conservation of energy to determine the final internal energy. Finally, we go to the refrigerant tables and find the T and P.

We can go to the refrigerant tables and find

$$u_1 = 212.18 \text{ kJ/kg} \quad T_1 = -26.43^\circ\text{C} \quad v_1 = 0.1917 \text{ m}^3/\text{kg}$$

Since our process is isotropic, we have

$$v_2 = v_1 = 0.1917 \text{ m}^3/\text{kg}$$

The final internal energy is given by the 1st law or

$$u_2 = q + u_1 = 66 + 212.18 = 278.18 \text{ kJ/kg}$$

So now we must go to the refrigerant tables and find the T and P that will correspond to these values of u and v. Since we had saturated vapor and we added heat, we will assume that we have gone to superheated vapor. Then scanning the tables we find that at 0.14 MPa and 60°C, we have

$$u = 278.74 \text{ kJ/kg} \quad v = 0.1902 \text{ m}^3/\text{kg}$$

which is close enough.

5. Consider a 300 gallon hot water heater which is to provide water a 180°F and 20 psia. If a typical shower consumes 3 gallons/minute, last 15 minutes, and requires that the hot water should stay above 160°F, determine the heat transfer rate required. Water is supplied at 63°F and 22 psia and 2 gallons/minute.

**Solution:**

System Type: Transient System

Substance Type: Incompressible

Process: Isotropic

Initial State: Fixed

Final State: Fixed

Inlet State: Fixed

Exit State: Fixed

Q = UNKNOWN

$$W_{sh} = 0$$

$$W_{bnd} = 0$$

$$\text{Conserv. of Mass: } \frac{m_2 - m_1}{\Delta t} = \dot{m}_{in} - \dot{m}_{out}$$

$$\text{1st Law: } \frac{m_2 u_2 - m_1 u_1}{\Delta t} = \dot{m}_{in} h_{in} - \dot{m}_{out} h_{out} + \dot{Q}$$

State 1	State In	State Out	State 2
$T_1 = 180^\circ\text{F}$	$T_{in} = 63^\circ\text{F}$	$T_{out} = 170^\circ\text{F}$	$T_2 = 160^\circ\text{F}$
$P_1 = 20 \text{ psia}$	$P_{in} = 22 \text{ psia}$	$P_{out} = 20 \text{ psia}$	$P_2 = 20 \text{ psia}$
$u_1 = 147.97 \text{ Btu/lb}_m$	$h_{in} = 31.43 \text{ Btu/lb}_m$	$h_{out} = 138.01 \text{ Btu/lb}_m$	$u_2 = 127.94 \text{ Btu/lb}_m$
$v_1 = 0.01651 \text{ ft}^3/\text{lb}_m$	$v_{in} = 0.01606 \text{ ft}^3/\text{lb}_m$	$v_{out} = 0.01645 \text{ ft}^3/\text{lb}_m$	$v_2 = 0.01640 \text{ ft}^3/\text{lb}_m$
$m_1 = 51.08 \text{ lb}_m$	$m_{in} = 0.0055 \text{ lb}_m/\text{s}$	$m_{out} = 0.0083 \text{ lb}_m/\text{s}$	$m_2 = \mathbf{48.56 \text{ lb}_m}$

Note that we have used the average temperature at state 1 and 2 to fix our outlet temperature

*Italicized* values are from steam tables. **Bold** values are calculated.

Approach: We begin by evaluating the properties at all states by using the steam tables. We can then use the conservation of mass to determine the mass at state 2. Finally the 1st law is used to determine the heat transfer rate.

Even though we will treat water as an incompressible substance in this case, we can use the steam tables to determine the specific volume and internal energy since for an incompressible substance they only depend on temperature, we can take the values for saturated liquid at the given temperature. For the enthalpy, we can use the enthalpy of saturated liquid and then bring it up to the appropriate pressure or

$$h_{\text{incompressible sub.}} = h_f + v_f(P - P_{\text{sat}})$$

Our values are then entered on the table. To obtain the masses and mass flows we convert gallons to  $\text{ft}^3$  and divide by the specific volume. These values are entered on the table. The final state mass is given by conservation of mass or

$$m_2 = \Delta t(\dot{m}_{\text{in}} - \dot{m}_{\text{out}}) + m_1 = (15)(60)(0.055 - 0.0083) + 51.08 = 48.56 \text{ lb}_m$$

Then the heat transfer rate becomes

$$\begin{aligned} \dot{Q} &= \frac{m_2 u_2 - m_1 u_1}{\Delta t} - \dot{m}_{\text{in}} h_{\text{in}} + \dot{m}_{\text{out}} h_{\text{out}} \\ &= \frac{(48.56)(127.94) - (51.08)(147.97)}{(15)(60)} \\ &\quad - (0.0055)(31.43) + (0.0083)(138.01) \\ &= -0.522 \text{ Btu/s} = -1881 \text{ Btu/hr} \end{aligned}$$

6. A piston cylinder system contains gas at 2300 K, 2500 kPa, and 0.03 liters. The gas then undergoes a polytropic expansion with a polytropic exponent of 1.15 to 0.3 liters. Compare the work performed in kJ for air as the gas versus hydrogen as the gas.

*Solution:* We start this problem by working in air and then in hydrogen

System Type: Closed System

Substance Type:: Ideal gas

Process: Polytropic with  $n=1.15$

Initial State: Fixed

Final State: UNKNOWN

$Q = \text{UNKNOWN}$  (but not asked for)

$$W_{\text{sh}} = 0$$

$W_{\text{bnd}} = \text{UNKNOWN}$

Conservation of Mass:  $m_1 = m_2$

1st Law:  $m(u_2 - u_1) = Q - W_{\text{bnd}}$

$$\text{Boundary Work: } W_{\text{bnd}} = \frac{P_2 v_2 - P_1 v_1}{(1 - n)}$$

<b>State 1</b>	<b>State 2</b>
$T_1 = 2300 \text{ K}$	$T_2 = \text{NA}$
$P_1 = 2500 \text{ kPa}$	$P_2 = \mathbf{162.8 \text{ kPa}}$
$V_1 = 3 \times 10^{-5} \text{ m}^3$	$V_2 = 3 \times 10^{-4} \text{ m}^3$

*Italicized* values are from ideal gas relations. **Bold** values are calculated.

Approach: At state 2 we can determine the pressure from the polytropic relation. We can then use our boundary work equation to calculate the boundary work

From the polytropic relationship we have

$$P_2 V_2^n = P_1 V_1^n$$

or solving for  $P_2$

$$P_2 = P_1 \left( \frac{V_1}{V_2} \right)^n = (2300) \left( \frac{0.03}{0.3} \right)^{1.15} = 162.8 \text{ kPa}$$

The boundary work is then given by

$$W_{\text{bnd}} = \frac{(162.8)(3 \times 10^{-4}) - (2300)(3 \times 10^{-5})}{(1-1.15)} = 0.134 \text{ kJ}$$

Since the calculation above never used the fact that air was our ideal gas, we will obtain the same boundary work for hydrogen as the gas.

7. In an open feedwater, subcooled liquid water is heated to saturated liquid by mixing directly with steam. In a given situation water at 10 MPa and 200°C enters an open feedwater heater at 12 kg/s. Steam at 10 MPa and 350°C is available to be added to this water to produce saturated liquid at 10 MPa. How much steam in kg/s must be added?

**Solution:**

System Type: Control Volume System

Substance Type: Water (compressible substance)

Process: Isobaric

Inlet State: Fixed

Exit State: Fixed

$$Q = 0$$

$$W_{\text{sh}} = 0$$

$$\text{Conserv. of Mass: } \dot{m}_{\text{in-1}} + \dot{m}_{\text{in-2}} - \dot{m}_{\text{out}} = 0$$

$$\text{1st Law: } \dot{m}_{\text{in-1}} h_{\text{in-1}} + \dot{m}_{\text{in-2}} h_{\text{in-2}} - \dot{m}_{\text{out}} h_{\text{out}} = 0$$

<b>State In-1</b>	<b>State In-2</b>	<b>State Out</b>
$T_{\text{in-1}} = 200^\circ\text{C}$	$T_{\text{in-2}} = 350^\circ\text{C}$	$T_{\text{out}} = 311^\circ\text{C}$
$P_{\text{in-1}} = 10 \text{ MPa}$	$P_{\text{in-2}} = 10 \text{ MPa}$	$P_{\text{out}} = 10 \text{ MPa}$
$h_{\text{in-1}} = 856 \text{ kJ/kg}$	$h_{\text{in-2}} = 2923.4 \text{ kJ/kg}$	$h_{\text{out}} = 1407.56 \text{ kJ/kg}$
$\dot{m}_{\text{in-1}} = 12 \text{ kg/s}$	$\dot{m}_{\text{in-2}} = \mathbf{4.366 \text{ kg/s}}$	$\dot{m}_{\text{out}} = \mathbf{16.366 \text{ kg/s}}$
phase: <i>sub.liq.</i>	phase: <i>sub.liq.</i>	phase: <i>sat.liq.</i>

*Italicized* values are from steam tables. **Bold** values are calculated.

Approach: We begin by evaluating the properties at all states by using the steam tables. We can then use the conservation of mass to algebraically solve for the exit mass flow rate. Substituting this into the 1st law steam flow rate can be determined.

The enthalpies are read from the steam tables and entered into our table above.

Solving for  $\dot{m}_{\text{out}}$  from conservation of mass

$$\dot{m}_{\text{out}} = \dot{m}_{\text{in-1}} + \dot{m}_{\text{in-2}}$$

Substituting into the energy equation

$$\dot{m}_{\text{in-1}}h_{\text{in-1}} + \dot{m}_{\text{in-2}}h_{\text{in-2}} - (\dot{m}_{\text{in-1}} + \dot{m}_{\text{in-2}})h_{\text{out}} = 0$$

Solving for  $\dot{m}_{\text{in-2}}$

$$\dot{m}_{\text{in-2}} = \frac{\dot{m}_{\text{in-1}}(h_{\text{in-1}} - h_{\text{out}})}{h_{\text{out}} - h_{\text{in-2}}} = \frac{12(856 - 1407.56)}{1407.56 - 2923.4} = 4.366 \text{ kg}$$

8. Most of the time during the winter Dr. Somerton turns down the thermostat to 50°F when he leaves in the morning. When he is in the house he likes to have the temperature at 68°F. The house may be considered to be composed of air, occupying a volume of 10,000 ft<sup>3</sup>, and structural material (mostly wood) of 11,000 lb<sub>m</sub>. Determine the total heat transfer required to bring the house up to 68°F. What fraction of this total goes to heating up the air and what fraction goes to heating up the structural material?

*Solution System Type:* Closed System

Substance Type: Ideal gas (air) and Incompressible (wood)

Process: Isobaric

Initial State: Fixed

Final State: Fixed

Q = UNKNOWN

$$W_{\text{sh}} = 0$$

$$W_{\text{bnd}} = 0$$

Conservation of Mass:  $m_1 = m_2$

1st Law:  $U_2 - U_1 = Q$

<b>State 1</b>	<b>State 2</b>
$T_1 = 50^\circ\text{F}$	$T_2 = 68^\circ\text{F}$
$V_{1,\text{a}} = 10,000 \text{ ft}^3$	$V_{2,\text{a}} = 10,000 \text{ ft}^3$
$m_{1,\text{a}} = 779 \text{ lb}_m$	$m_{2,\text{a}} = 779 \text{ lb}_m$
$m_{1,\text{w}} = 11,000 \text{ lb}_m$	$m_{2,\text{w}} = 11,000 \text{ lb}_m$

*Italicized* values are from ideal gas relations. **Bold** values are calculated.

Approach: We begin by determining the mass of air in the house. Then we can use the 1st law to calculate the heat transfer required.

We determine our mass of air assuming a pressure of 14.7 psia. Using the ideal gas law we have

$$m_{1,\text{a}} = \frac{P_1 V_{1,\text{a}}}{RT_1} = \frac{(14.7)(10,000)}{(10.73/29)(510)} = 779 \text{ lb}_m$$

We can calculate the heat transfer from the energy equation

$$Q = U_2 - U_1 = m_a c_{v,a} (T_2 - T_1) + m_w c_{P,w} (T_2 - T_1)$$

$$= (779)(0.171)(68 - 50) + (11,000)(0.42)(68 - 50) = 85,558 \text{ Btu}$$

of which 97% goes into the structural material and 3% into the air.

9. Oil enters an ideal pump at 100°F and 12 psia and leaves at 17 psia. The oil flow rate is 0.04 lb<sub>m</sub>/s. The pump inlet has a diameter of 5 inches and the pump outlet has a diameter of 2 inches. What pumping power is required?

**Solution:**

System Type: Control Volume System

Substance Type: Incompressible

Process: Isentropic, can't neglect KE

Initial State: Fixed

Final State: Unknown

$$Q = 0$$

$$W_{sh} = \text{UNKNOWN}$$

$$\text{Conservation of Mass: } \dot{m}_2 = \dot{m}_1 = 0.04 \text{ lb}_m/\text{s}$$

$$\text{1st Law: } \dot{m} \left( h_2 + \frac{\bar{v}_2^2}{2} - h_1 - \frac{\bar{v}_1^2}{2} \right) = -\dot{W}_{sh}$$

<b>State 1</b>	<b>State 2</b>
T <sub>1</sub> = 100°F	T <sub>2</sub> = <b>100°F</b>
P <sub>1</sub> = 12 psia	P <sub>2</sub> = 17 psia
$\bar{v}_1 = \mathbf{0.005 \text{ ft/s}}$	$\bar{v}_2 = \mathbf{0.032 \text{ ft/s}}$

**Bold** values are calculated.

Approach: We begin by using our process description to fix state 2. Then from the diameter and mass flow information the velocities are obtained. Finally, the 1st law is used to calculate the power.

Since the process is isentropic and we have an incompressible substance, we write

$$s_2 - s_1 = c_{P,avg} \cdot \ln \left( \frac{T_2}{T_1} \right)$$

which yields

$$T_2 = T_1 = 100^\circ\text{F}$$

The velocities can be calculated from one of our continuity relations

$$\bar{v} = \frac{\dot{m}}{\rho \cdot A_c} = \frac{\dot{m}}{\rho \cdot \frac{\pi D^2}{4}}$$

Using a density for oil of  $57 \text{ lb}_m/\text{ft}^3$ , we find

$$\bar{v}_1 = 0.005 \text{ ft/s}$$

$$\bar{v}_2 = 0.032 \text{ ft/s}$$

The enthalpy change for an incompressible substance is given by

$$h_2 - h_1 = c_{P,\text{avg}}(T_2 - T_1) + v_{\text{avg}}(P_2 - P_1)$$

Then the shaft work is calculated (using the density rather than the specific volume)

$$\begin{aligned} \dot{W}_{\text{sh}} &= -\dot{m} \left( \frac{P_2 - P_1}{\rho} + \frac{\bar{v}_2^2 - \bar{v}_1^2}{2} \right) \\ &= -(0.04) \left( \frac{17 - 12}{57} \cdot \left[ \frac{1}{5.404} \frac{\text{Btu}}{\text{psia} \cdot \text{ft}^3} \right] + \frac{(0.032)^2 - (0.005)^2}{2} \cdot \left[ \frac{1}{25.037} \frac{\text{Btu}/\text{lb}_m}{\text{ft}^2/\text{s}^2} \right] \right) (3600 \text{ s/hr}) \\ &= -2.338 \text{ Btu/hr} \end{aligned}$$

10. The exhaust process for an internal combustion engine may be modeled as transient system undergoing an isobaric process with boundary work. Just before the exhaust valve opens the cylinder of 0.5 liters contains air at 200 kPa and 500 K. At the end of exhaust the volume is 0.0833 liters. Assume that the process is adiabatic. Determine the final temperature and mass and the boundary work.

**Solution:**

System Type: Transient System

Substance Type: Ideal gas

Process: Isobaric

Initial State: Fixed

Outlet State: UNKNOWN

Final State: UNKNOWN

$$Q = 0$$

$$W_{\text{sh}} = 0$$

$$W_{\text{bnd}} = P(V_2 - V_1)$$

$$\text{Conserv. of Mass: } m_2 - m_1 = -m_{\text{out}}$$

$$\text{1st Law: } m_2 u_2 - m_1 u_1 = -m_{\text{out}} h_{\text{out}} - W_{\text{bnd}}$$

<b>State 1</b>	<b>State Out</b>	<b>State 2</b>
$T_1 = 500 \text{ K}$	$T_{\text{out}} =$	$T_2 =$
$P_1 = 200 \text{ kPa}$	$P_{\text{out}} = 200 \text{ kPa}$	$P_2 = 200 \text{ kPa}$
$V_1 = 5 \times 10^{-4} \text{ m}^3$	$V_{\text{out}} = \text{NA}$	$V_2 = 8.33 \times 10^{-5} \text{ m}^3$

**Bold** values are calculated.

Approach: We begin by calculating the boundary work. We then use the ideal gas law and constant specific heat to determine the enthalpy and internal energies.

The boundary work is then

$$W_{\text{bnd}} = P(V_2 - V_1) = (200)[8.33 \times 10^{-5} - 5 \times 10^{-4}] = -0.0833 \text{ kJ}$$

Now we want to solve for the final temperature, but we note that in the first law four different things depend on  $T_2$ :  $u_2$ ,  $m_2$ ,  $h_{\text{out}}$ , and  $m_{\text{out}}$ . We can eliminate  $m_{\text{out}}$  with the help of our conservation of mass so that

$$m_2 u_2 - m_1 u_1 = - (m_1 - m_2) h_{\text{out}} - W_{\text{bnd}}$$

Since the exhaust air temperature is changing during the process, we will use our linear average approach to determine  $h_{\text{out}}$ , or

$$h_{\text{out}} = h_{\text{air}} \left( \text{at } \frac{T_1 + T_2}{2} \right)$$

The mass at the final state will be given by

$$m_2 = \frac{P_2 V_2}{RT_2}$$

So substituting

$$u_2 = \frac{RT_2}{P_2 V_2} \left[ m_1 u_1 - \left( m_1 - \frac{P_2 V_2}{RT_2} \right) h_{\text{air}} \left( \text{at } \frac{T_1 + T_2}{2} \right) - W_{\text{bnd}} \right]$$

When we consider that the air tables provide the relationships among  $u$  and  $T$  and  $h$  and  $T$ , we see that we do have a well posed problem, but we can't do the algebra, so we will have to solve the problem by iteration. Our process will be

Guess  $T_2$

Evaluate  $h_{\text{air}}$  and  $u_2$  from the air tables

Calculate  $u_2$  from the above equation

Compare the  $u_2$  at the guessed value of  $T_2$  to the calculated value of  $u_2$

Re-guess  $T_2$  and repeat the process until the difference between the values of  $u_2$  at the guessed  $T_2$  and the value of the calculated  $u_2$  is negligible

I set this up in an Excel spreadsheet and the results are shown below

$T_{2,\text{guess}}$	$u_{2,\text{guess}}$	$T_{\text{out}}$	$h_{\text{out}}$	$m_2$	$u_2$	$u_{2,\text{guess}} - u_{2,\text{calc}}$
500	147.31	500	205.28	0.000117	572.3208	-425.0108163
525	165.99	512.5	218.19	0.000111	522.2173	-356.2273109
550	184.82	525	231.14	0.000106	464.1406	-279.3206002
600	222.95	550	257.14	9.71E-05	324.0476	-101.097563
650	261.71	575	283.3	8.96E-05	151.6535	110.0564586
625	242.25	562.5	270.2	9.32E-05	241.9062	0.343817527

So it would appear that our final temperature is about 625 K.