ME 201
Thermodynamics

Final Exam Solutions

<u>Directions:</u> Work all three problems. The exam is open notes and open text book. All problems have equal weight. Note that you may round where appropriate to avoid interpolation.

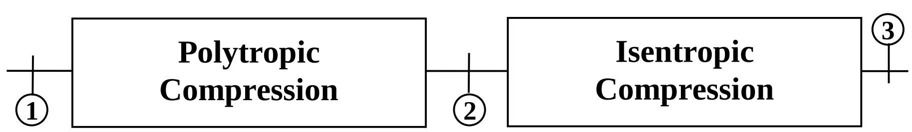
<u>Problem 1</u>
A more realistic model for the compression process in an internal combustion uses a polytropic process followed by an isentropic process instead of a single isentropic process. The engine operates with the following conditions:

Six cylinder, four stroke engine with displacement of 2.8 liters
Compression ratio: 9.065
Compression ratio for polytropic process: 3
Polytropic exponent for first compression process: 1.203
Initial air conditions: 280 K and 120 kPa
Engine speed of 1300 rpm

Determine

a.) the air temperature at the end of both compression processes
b. the heat transfer for the polytropic process
b.) the engine power in kW for the two compression processes

<u>Solution:</u>
We begin with a block diagram of the system nodded out for the two processes of interest.



Next we need to obtain our volumes. We are given

$V_{disp} = 2.8 \times 10^{-3}\ m^3$ and r = 9.065

then

$$V_{disp} = V_{BDC} - V_{TDC}$$
$$V_{BDC} = rV_{TDC}$$
$$V_{BDC} / V_2 = 3$$

Substituting gives

$$V_{TDC} = \frac{V_{disp}}{r - 1} = \frac{2.8x10^{-3}}{9.065 - 1} = 3.4718 \text{ x } 10^{-4} \text{ m}^3$$

and

$$V_{BDC} = (9.065)(3.4718 \text{ x } 10^{-4}) = 3.1472 \text{ x } 10^{-3} \text{m}^3$$

Following the polytropic process we have

$$V_2 = V_{BDC} / 3 = (3.14724 \text{ x } 10^{-3}) / 3 = 1.0491 \text{ x } 10^{-3} \text{ m}^3$$

Now setting up our table and entering our operating information

Node	T(K)	P(kPa)	V(m^3)	v(m^3/kg)	u(kJ/kg)	v_r
1	280	120	3.1472 x 10^{-3}	*0.6697*	*199.75*	
2	*350*	**449.94**	1.0491 x 10^{-3}	**0.2232**	*250.02*	*422.2*
3	*540*	*2098*	3.4718 x 10^{-4}	**0.0739**	*389.34*	**139.72**

Italicized values from ideal gas relations. **Bold** values calculated.

We begin by calculating our specific volume at 1

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287)(280)}{120} = 0.6697 \text{ m}^3 / \text{kg}$$

Our remaining properties can be read from the air tables. Next we calculate our mass

$$m = \frac{V_{BDC}}{v_1} = \frac{3.1472 \text{ x } 10^{-3}}{0.6697} = 4.70x10^{-3} \text{ kg}$$

Then the remaining specific volumes are calculated using

$$v = \frac{V}{m}$$

Now traversing the cycle.

1-2 Polytropic Expansion

$$P_2 = P_1 \left(\frac{v_1}{v_2} \right)^n = (120) \left(\frac{0.6697}{0.2232} \right)^{1.203} = 449.94 \text{ kPa}$$

The temperature is given by the ideal gas law

$$T_2 = \frac{P_2 v_2}{R} = \frac{(449.94)(0.2232)}{(0.287)} = 350 \text{ K}$$

To get the remaining properties we go to the air table.

2-3 Isentropic expansion

We will use

$$\frac{v_{r3}}{v_{r2}} = \left(\frac{v_3}{v_2} \right)_{isentropic}$$

to fix our state at 3, or solving

$$v_{r3} = v_{r2}\left(\frac{v_3}{v_2}\right) = (422.2)\frac{0.0739}{0.2232} = 139.72$$

which from the air tables gives us a T_3 of 540 K with the remaining properties entered on the table. The pressure is calculated form the ideal gas law

$$P_3 = \frac{RT_3}{v_3} = \frac{(0.287)(540)}{0.0739} = 2098\,\text{kPa}$$

The work output is given by

$$W_{comp} = W_{1-2} + W_{2-3}$$

where

$$W_{1-2} = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{(449.94)(1.0491\,\text{x}\,10^{-3}) - (120)(3.1472\,\text{x}\,10^{-3})}{1-1.203}$$

$$= -0.4648\ \text{kJ}$$

$$W_{2-3} = m(u_2 - u_3) = (4.70\,\text{x}\,10^{-3})(250.03 - 379.34) = -0.6548\ \text{kJ}$$

which gives

$$W_{comp} = -1.1196\ \text{kJ}$$

The power is then given by

$$\dot{W}_{comp} = |W_{comp}|(\text{rps})(2/N_{strokes}) = |-1.1196|(1300/60)(2/4)$$

$$= 12.13\,\text{kW}$$

The heat transfer for the polytropic process is given by

$$Q_{1-2} = m(u_2 - u_1) + W_{1-2} = (4.70\,\text{x}\,10^{-3})(250.03 - 199.75) + (-0.4648)$$

$$= -0.2286\ \text{kJ}$$

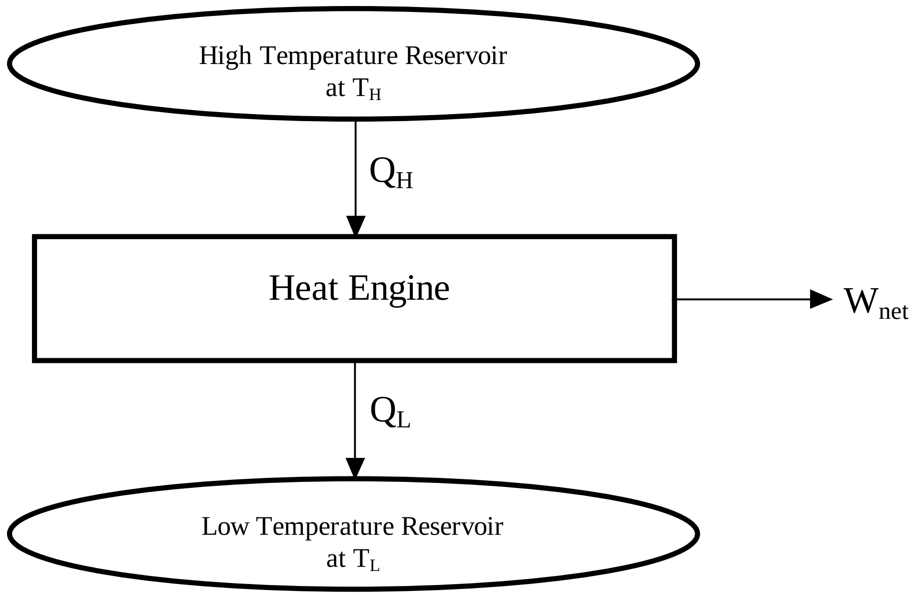
Problem 2

It has been found that microwave heating of methane gas increases its combustion temperature from 2700 K to 3000 K. We wish to determine if the use of microwave heating is cost effective. Consider a power plant that is producing 1 MW of electric power as it operates with a low temperature of 290 K. The electric power sells for and costs \$0.04 per kW·hr, and the heat transfer from the high temperature reservoir costs \$0.01 per kW·hr. To achieve the 300 K increase in combustion temperature, 50 kW of electric power must be used. Determine

(a) maximum income per year the plant can generate without the microwave heating
(b) maximum income per year the plant can generate with the microwave heating

Solution:

WE have twp power plants to compare, one with a high temperature heat reservoir of 2700 K and a second with a high temperature heat reservoir of 3000 K. We begin with our interaction diagram for a heat engine.



Our appropriate equations are:

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_H}$$

$$\dot{Q}_H = \dot{W}_{net} + \dot{Q}_L$$

$$\eta_{Carnot} = 1 - \frac{T_L}{T_H}$$

Our income is given by

Income = (Electric Energy Generated)(0.04) - (Heat Energy Consumed)(0.0075)
- (Electric Energy Consumed)(0.04)

where

Electric Energy Generated = $\dot{W}_{net}$ x # of hours in a year
= (1000 kW)(24 x 365) = 8,760,000 kW·hr

Heat Energy Consumed = $\dot{Q}_H \times$ # of hours in a year

We can determine $\dot{Q}_H$ from our thermal efficiency. Since we are asked for the maximum incomes the thermal efficiency will be calculated from our Carnot equation. Starting with the non-microwave heating plant we have or

$$\eta_{Carnot} = 1 - \frac{290}{2700} = 0.893$$

Our heat transfer rate is then

$$\dot{Q}_H = \frac{\dot{W}_{net}}{\eta_{th}} = \frac{1000}{0.893} = 1120\,\text{kW}$$

and our annual income is

Income = (8,760,000)(0.04) - (1120)(24)(365)(0.01) = $252,259

Now for the microwave heating plant

$$\eta_{Carnot} = 1 - \frac{290}{3000} = 0.903$$

Our heat transfer rate is then

$$\dot{Q}_H = \frac{\dot{W}_{net}}{\eta_{th}} = \frac{1000}{0.903} = 1107\,\text{kW}$$

and our annual income is

Income = (8,760,000)(0.04) - (1107)(24)(365)(0.01)
- (50)(24)(365)(0.04) = $235,906

So the microwave heating is not economically viable.

Problem 3

Steam enters the compressor with isentropic efficiency 0.7626 at 50°C and 0.01 MPa and exits at 800 kPa. Determine

a.) the ideal work input (in kJ/kg) required
b.) the actual work input (in kJ/kg) required
c.) the exit temperature of the steam

Solution:

The actual work output for this compressor will be calculated from

$$w_{act} = \frac{w_{ideal}}{\eta_s}$$

so that we will need to do an ideal 1st law turbine calculation.

System Type: Control Volume
Substance Type: Compressible (steam)
Device: Non-Ideal Compressor(adiabatic with η_s = 0.7626)
State 1 (steam inlet): Fixed
State 2s (ideal steam outlet): unknown
State 2a (actual steam outlet): unknown
W_{sh}: unknown
Q = 0
W_{bnd} = 0
conservation of mass: $\dot{m}_1 = \dot{m}_2$
1st law: $(h_1 - h_{2s}) = w_{ideal}$, $(h_1 - h_{2a}) = w_{act}$

State 1 (inlet)	State 2s (ideal outlet)	State 2a (actual outlet)
T_1 = 50°C	T_{2s} = *600°C*	T_{2a} = *752°C*
P_1 = 0.01 MPa	P_{2s} = 800 kPa	P_{2a} = 800 kPa
h_1 = *2592.0 kJ/kg*	h_{2s} = *3700.1 kJ/kg*	h_{2a} = **4045.1 kJ/kg**
s_1 = *8.1741 kJ/(kg K)*	s_{2s} = **8.1741 kJ/(kg K)**	s_{2a} =
phase: *sup.vap*	phase: *sup.vap*	phase: *sup.vap*

Italicized values from tables. **Bold** values are calculated

Approach: Since state 1 is fixed we may go to the steam tables and obtain the remaining properties. We may fix state 2s by using the isentropic condition for the ideal turbine, and then use the 1st law to calculate the ideal power. The actual power can be calculated from the efficiency equation, and then the actual exit enthalpy from the first law. This will fix state 2a, and allow us to determine the temperature.

At 0.01 MPa and 50°C we have superheated vapor. Going to the superheat tables we find

h_1 = 2592.0 kJ/k and s_1 = 8.1741 kJ/(kg K)

At state 2s we have

$s_{2s} = s_1$ = 8.1741 kJ/(kg K)

which then fixes the state at 2s. From the steam tables we find at 800 kPa

T_{2s} = 600°C and h_{2s} = 3700.1 kJ/kg

The ideal work is then calculated from the 1st law as

$$w_{ideal} = (h_1 - h_{2s}) = (2592.0 - 3298.7) = -1108.1 \text{ kJ/kg}$$

The actual work is calculated from the efficiency

$$w_{act} = \frac{w_{ideal}}{\eta_s} = \frac{-1108.1}{0.7626} = -1453.1 \, \text{kJ/kg}$$

The actual exit enthalpy is calculated from the 1st law

$$h_{2a} = h_1 - w_{act} = 2592.0 - (-1453.1) = 4045.1 \text{ kJ/kg}$$

Interpolating to find the temperature we have

T_{2a} = 752°C